LINEAR MODEL OF A CLOSED THREE-SHAFT BRAYTON CYCLE

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Abstract: A linear model of a closed three-shaft Brayton cycle at normal power operation is developed. The model is intended to give an understanding of the dominant dynamic behaviour in a nuclear power system that utilizes a closed three-shaft Brayton cycle. The insights gained from the model can be used for system design as well as for the design of control algorithms.
A conceptual linear model is constructed and it is explained how this model can be developed starting with the models of the turbines and compressors followed by models of the shafts and the volumes inside the circuit to finally obtain the linear state space equations. A time domain comparison is made between the linear system model and the responses predicted by the thermal-fluid analysis software program, Flownet.

Key Words. Linear model, dominant dynamic behaviour, control design, Brayton cycle, Flownet

1. INTRODUCTION

The first replica of a three-shaft Brayton cycle was built at the North-West University in Potchefstroom, South Africa to test some of the design concepts of a nuclear power plant called the Pebble Bed Modular Reactor (PBMR) also being developed in South Africa [1]. Although the PBMR has changed their design to a single-shaft system, the single-shaft system is not used here for reasons of confidentiality. The method developed can however easily be applied on the single-shaft design.

Figure 1 shows the layout of the three-shaft PBMR. Starting at the reactor outlet, helium at a high temperature and pressure is expanded in a high-pressure turbine (HPT), low-pressure turbine (LPT) and power turbine (PT). It is then cooled in the recuperator and pre-cooler and is then compressed by the compressors (LPC and HPC) with inter-cooling to improve efficiency. The helium is then preheated in the recuperator before entering the reactor [2].

In the start-up transition prior to normal power operation, the reactor is already heated up and the power turbine is externally maintained at 50 Hz. A start-up blower system (SBS) is used to provide initial flow in the cycle. Although enough helium is already in the system, much of it is bypassed through the compressors with the low pressure bypass and gas bypass control valves (LPB and GBPC) both open. These valves are then systematically closed to allow the HPT and LPT to be run up through their critical frequency ranges (between 20% and 80% of nominal speed). During this process, the Brayton cycle becomes self sustained and the SBS can be deactivated.

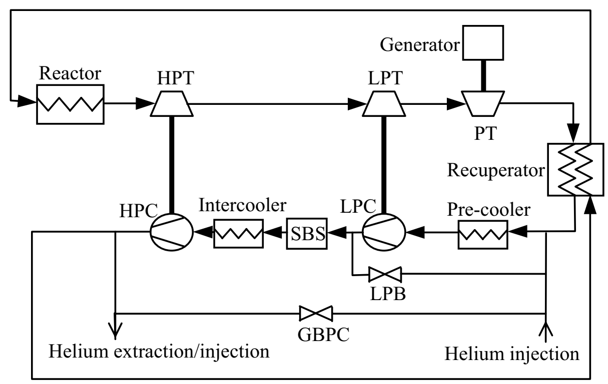

Figure 1. Layout of the 3-shaft PBMR.

During normal power operation, assuming that all bypass valves are closed, the amount of power delivered by the power station would be proportional to the amount of helium in the system. Helium is extracted at the HP compressor outlet and is normally injected at the pre-cooler inlet whilst it can also be injected at the HP compressor outlet via a booster tank if the pressure in the booster tank allows it. Faster responses can be obtained by allowing bypass flow through the compressors via the GBPC, providing reserve power that can be used during sudden demand changes.

This paper discusses the development of a simplified linear model of the system at normal power operation.

The model captures the dominant dynamic behaviour of the system and consists of simplified equations that can be linked to physical parameters such as shaft inertias and volume sizes. White box models such as this are needed especially at the beginning of a complex system's design phase to aid the control system design process [3] and facilitate discussions and design choices surrounding the selection and sizing of system components [4].

The model is validated against a Flownet model of the system. Flownet (currently known as Flownex®) is a user-friendly software package allowing the dynamic simulation of mass, momentum and energy transfer in thermal-fluid networks integrated with controllers. It is currently used extensively in the modeling and design of the PBMR [5].

2. CONSTRUCTION OF THE SIMPLIFIED MODEL

Within the system various non-linear relationships exist between the system parameters such as temperatures, pressures and shaft speeds. The shaft inertias and volumes in the system represent energy storage elements. The relationship between the parameters and the effect of the volumes and shaft inertias will depend on the particular mode in which the power system is. This paper concentrates on the normal power operation mode of the power system and for the purpose of the linear model the following assumptions are made:

- The dynamics of the reactor are much slower than the dynamics of the rest of the system. The outlet temperature of the reactor is therefore assumed to be constant.
- The inlet temperatures to the compressors remain constant during small transients due to the presence of the pre-cooler and intercooler.
- The dynamic changes in pressure losses around the circuit (except for those inherent in the turbo machines) are considered to have a negligible effect on the system dynamics.
- The leak flows between components have negligible effects during transients.
- The inlet, outlet and bypass valves can be modeled as mass flow sources.
- It is assumed that the effects of the system between the turbo machines (turbines and compressors), can be modeled as lumped volumes or fluid capacitances.
- It is assumed that the generator is locked to the power grid and as such, the power turbine shaft speed is fixed.

With all of these assumptions, the simplified model of the PCU is structured according to the sketch in Figure 2 which is partly drawn as an electric circuit where the electric capacitance, current and voltage would be analogous to the fluid capacitance, mass flow rate and pressure respectively.

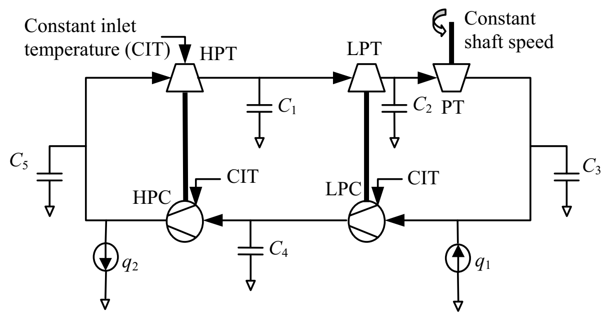

Figure 2. Conceptual model at normal power operation

For control purposes, the interest is in the system response due to perturbations in the steady state mass flow rates. The model therefore makes use of the partial derivatives of the system differential equations in which the steady state values are substituted as will be described in the next section. The mass flow rates of the injection, extraction and bypass are therefore modeled as the perturbations q_1 and q_2 and not the actual flow rates Q_1 and Q_2.

The energy storages are firstly the HP and LP shafts and secondly the volumes in the system adding up to a total of seven. Equation 1 describes the relationship between the shaft speed n, the shaft inertia J and the compressor and turbine torques τ_c and τ_t.

$$n(t) = \frac{1}{J}\int\left(\tau_c(t) + \tau_t(t)\right)dt \qquad (1)$$

Equation 2 describes the relationship between the pressure p, the fluid capacitance C and the inlet and outlet flow rates q_{in} and q_{out}.

$$p(t) = \frac{1}{C}\int\left(q_{in}(t) - q_{out}(t)\right)dt \qquad (2)$$

3. LINEAR MODEL DEVELOPMENT

Since the objective is to develop a linear model that will give insight into the dynamics of the power control unit, attention is only given to the variations in the state variables due to perturbations in some of the other state variables, and not the absolute value of the system state variables. The partial derivatives of the system at a normal power operation operating point are used to develop a linear model of the system.

The input/output relationships of a turbine can be used to illustrate the concept. The inlet pressure, outlet pressure, inlet temperature and shaft speed are used to calculate the power output, mass flow rate and outlet temperature of the machine as shown in Figure 3.

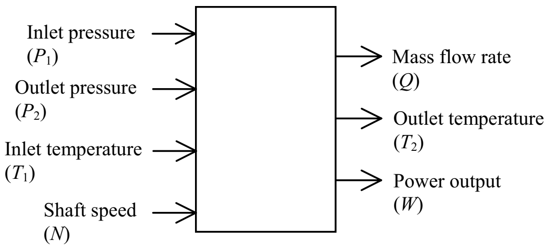

Figure 3. Input/output relationships used for turbo machines

The mass flow rate through a turbine is a function of the pressure ratio, the inlet pressure, the inlet temperature and the shaft speed. The pressure ratio in turn is a function of the inlet pressure and outlet pressure according to:

$$P_{rt} = P_1 / P_2 \tag{3}$$

The perturbation of the pressure ratio from the steady state pressure ratio at the given operating point can be described by:

$$p_{rt} = K_1 \cdot p_1 + K_2 \cdot p_2 \tag{4}$$

The K values are the partial derivatives:

$$\begin{aligned} K_1 &= \frac{\partial P_{rt}}{\partial P_1} = \frac{1}{P_{02}} \\ K_2 &= \frac{\partial P_{rt}}{\partial P_2} = -\frac{P_{01}}{P_{02}^2} \end{aligned} \tag{5}$$

The subscript 0 denotes the steady state quantity at the operating point. Similarly, the function for mass flow rate can be expressed as:

$$Q = f(P_{tr}, P_1, T_1, N) \tag{6}$$

And its perturbation from Q_0 as:

$$q = K_3 \cdot p_{tr} + K_4 \cdot p_1 + K_5 \cdot t_1 + K_6 \cdot n \tag{7}$$

The relationship between the mass flow rate, shaft speed and the pressure ratio is given by the turbo manufacturer's maps as will be discussed in the next section. Some partial derivatives therefore have to be calculated numerically via the maps. Combining the expressions for mass flow rate and pressure ratio results in:

$$q = K_3 \cdot (K_1 \cdot p_1 + K_2 \cdot p_2) + K_4 \cdot p_1 + K_5 \cdot t_1 + K_6 \cdot n \tag{8}$$

A similar procedure is used to determine the perturbed outlet temperature and power output.

4. MODELS OF THE TURBO-MACHINES

The turbine and compressor models (both non-linear and linear) presented here make the following basic assumptions:

- The machine has no inlet or outlet volume. The volume of diffusers or the inlet chamber must be accounted for using external volume elements.
- There is infinitely fast pressure transfer across the machine (i.e. there is no significant volume inside the turbine or compressor). This means that if the pressure on one side of the machine changes, the effect is immediately felt on the other side.
- The steady state turbine/compressor maps provided by the manufacturer are assumed to be valid under transient conditions. This is the same assumption made in Flownet and [6].
- There is no energy flow across the boundaries of the machine, except via the gas flow and the shaft.
- The shaft dynamics are not included as part of the turbine model, but are dealt with separately.

These assumptions indicate that the turbine models do not allow for any energy storage within the turbine itself. As such, the turbine merely presents an instantaneous mapping from input to output variables. Energy storage elements such as the shaft inertia and local volumes are accounted for by external models.

The turbine and compressor behaviour can largely be described by their steady state operating maps. Each machine can be described by 2 maps namely one that relates non-dimensional mass flow (Q') and non-dimensional speed (N') to pressure ratio (P_{rt}, P_{rc}) and another that relates non-dimensional speed and non-dimensional mass flow to isentropic efficiency (η_t, η_c). Examples of these maps are illustrated in Figure 4.

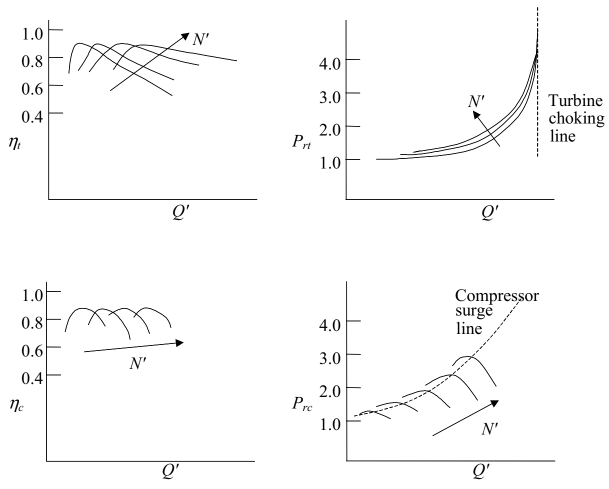

Figure 4. Typical shapes of the turbine and compressor maps.

The following relationships apply:

$$\begin{aligned} Q' &= Q\sqrt{T_1}\big/P_1 \\ N' &= N\big/\sqrt{T_1} \\ P_{rt} &= P_1/P_2 \\ P_{rc} &= P_2/P_1 \end{aligned} \tag{9}$$

Q is the mass flow rate through the machine (in kg/s), N is the speed of the machine shaft (in rev/s), T_1 is the machine inlet temperature (in K) and P_1 and P_2 are the machine inlet and outlet pressures respectively (in bar). Some observations regarding the turbo machine maps are appropriate:

- So-called "non dimensional" mass flow rate and actual mass flow rate must not be confused. In reality, there may typically be large changes in actual mass flow rate, but relatively small changes in non dimensional flow rate due to its dependence on T_1 and P_1.
- Actual shaft speed must not be confused with "non-dimensional" shaft speed. In particular, physical shaft speed may remain fixed (as in the case of a power turbine attached to a generator), but because of inlet temperature changes, non-dimensional speed may change.
- Since the mass flow rate through a turbine saturates as the pressure ratio is increased, a turbine typically operates substantially to the left of the choking line (Figure 4) for reasons of efficiency.
- The actual compressor operating point will, for safety reasons, always lie to the right of the surge line. Surging occurs when the mass flow rate is not enough to sustain the pressure gain. The flow direction is then reversed causing damage on the blades.

Normally, the pressure ratio, inlet temperature and shaft speed would be the known input variables from which the mass flow rate and isentropic efficiency can be determined via the maps. The outlet temperatures can be determined using the following relationships for turbines and compressors respectively:

$$\begin{aligned} T_2 &= T_1 - \eta_t T_1\left(1 - P_{rt}^{-\left(\frac{\gamma-1}{\gamma}\right)}\right) \\ T_2 &= T_1 - \frac{1}{\eta_c}T_1\left(1 - P_{rc}^{\left(\frac{\gamma-1}{\gamma}\right)}\right) \end{aligned} \tag{10}$$

γ is the ratio between the specific heat at constant pressure C_p and the specific heat at constant volume C_v. The power generated by the turbine or absorbed by the compressor can be determined by:

$$W = QC_p\left(T_1 - T_2\right) \tag{11}$$

The symbol W is used for power to not confuse power and pressure. The mechanical efficiency of the turbine is not included at this stage, but is included in the shaft model.Given these non-linear relationships a linear turbine model is derived as:

$$\begin{bmatrix} q \\ t_2 \\ w \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ n \\ t_1 \end{bmatrix} \tag{12}$$

The respective K values can be expressed in terms of the partial derivatives and other system constants as explained in Section 2. Equation 12 is only valid if the linearised parameters vary slowly with respect to the linearised closed loop system dynamics [7]. It is assumed that this condition is met in a steady state equilibrium point during normal power operation.

The efficiency changes in the turbine, under small transient conditions are generally so small as to negligibly affect the dynamic performance of the turbine. For this reason, no efficiency changes are allowed for here and efficiency is assumed to be a constant.

The procedure for developing a linear compressor model is the same as for the turbine model. As with the turbine model, it is assumed that the change in compressor efficiency is small during a transient, as to have negligible effect on the dynamics.

4. LINEAR MODEL OF THE TURBO-MACHINE SHAFT DYNAMICS

The power delivered by a turbine to its shaft is given by:

$$W_{shaft} = \eta_m W_t = 2\pi N T_t \tag{13}$$

W_t is the work done on the shaft by the turbine and η_m is the mechanical efficiency of the shaft, taking into account friction losses in both the turbine and the compressor. T_t is the torque supplied by the turbine to the shaft and N is the shaft speed in rev/s. Hence the torque supplied by the turbine to the shaft is given by:

$$T_t = \frac{\eta_m W_t}{2\pi N} \tag{14}$$

If it is assumed that the mechanical efficiency remains constant, the expression in Equation 14 can be linearised to find the torque perturbation due to a change in speed or shaft power:

$$\tau_t = \frac{\partial T_t}{\partial W_t} w_t + \frac{\partial T_t}{\partial N} n \tag{15}$$

The derivation of the compressor counter torque is similar to the turbine torque except for the omission of the mechanical efficiency term η_m which is accounted for only in the turbine equation. Recognizing that the net accelerating torque on the shaft is given by the sum of the torques acting on the shaft, the change in shaft speed can be obtained using Newton's second law:

$$n(t) = \frac{1}{2\pi J} \int \left(\tau_c(t) + \tau_t(t) \right) dt \tag{16}$$

J is the total spinning inertia of the shaft in Nm/rad/s. The factor 2π is introduced to convert speed from rad/s to rev/s (Hz). If it is recognized that in the steady state, the compressor and turbine powers are equal (because the shaft speed is steady), it follows that $W_{0c} = \eta_m W_{0t}$. Hence, the $\frac{\partial T}{\partial N} n$ terms cancel. Equation 16 can therefore be written as:

$$n(t) = \frac{1}{J'} \int \left(\frac{\partial T_c}{\partial W_c} \cdot w_c(t) + \frac{\partial T_t}{\partial W_t} \cdot w_t(t) \right) dt \tag{17}$$

Where $J' = 2\pi J$.

5. LINEAR MODEL OF THE VOLUMES IN THE SYSTEM

Buckley [8] shows that for fluids, a volume is analogous to a capacitance in an electric circuit, pressure is analogous to voltage and volumetric flow rate (or mass flow rate) is analogous to current. This relationship is especially true for perturbed variables (i.e. the steady state values have been removed). The capacitance can be written in the following form:

$$C = \frac{V}{nRT_0} \tag{18}$$

C is the system capacitance in kg/Pa, V is the volume in m^3, R is the gas constant and n is a constant which under adiabatic conditions equals the specific heat ratio γ of the gas, while under isothermal conditions (i.e. slowly varying changes in flow and pressure), $n = 1$ gives good results.

It follows that the dynamic model for a volume is given by:

$$p(t) = \frac{1}{C'} \int \left(q_{in}(t) - q_{out}(t) \right) dt \tag{19}$$

$C' = C \times 10^5$ converts the pressure from Pascal to bar.

6. SYSTEM EQUATIONS

With the use of the developed equations, the system equations can be derived in state space format. The resulting state space model has the familiar format:

$$\begin{aligned} \mathrm{X} &= \mathrm{AX} + \mathrm{BU} \\ \mathrm{Y} &= \mathrm{CX} \end{aligned} \tag{20}$$

Referring to Figure 2, q_1 and q_2 would be the variables of the input vector **U**. The pressures at the capacitors and the HP and LP shaft speeds are the state variables in **X**. The output vector **Y** comprises the flow rates, the turbo power values and the state variables of vector **X**.

7. COMPARISON BETWEEN THE LINEAR MODEL AND FLOWNET

In order to test the validity of the model, simulations were performed with helium injection, extraction and bypass and compared to similar simulations in Flownet (a commercial thermo-hydraulic simulation package). To limit the complexity of the Flownet model, the following assumptions were made for the normal power operating region:

- The reactor is modeled simply as a constant temperature node. Thus, the inlet temperature to the HPT is constant at 900 ^{o}C.
- The inlet temperature to the LPC is fixed at 28 ^{o}C.
- The inlet temperature to the HPC is fixed at 28 ^{o}C.
- All the pipes in the network have negligibly small pressure losses.

- The leak flows were all fixed to pre-defined values during the simulation. This was to ensure that no dynamic changes in leak-flow occurred during transients. The reason leak flows were included was to ensure that the turbo-machines operate at similar operating points on the pressure ratio and efficiency maps as compared to the PBMR v600 network.

Although moderate deviations occurred in some instances due to the fact that not all system dynamics were taken into account, the results showed that the model contains enough information to reflect the dominant dynamics of the system. Figures 5, 6 and 7 respectively show the flow rate, pressure and PT power output results on a helium injection simulation where mass was injected at a rate of 1 kg/s for a period of 100 seconds.

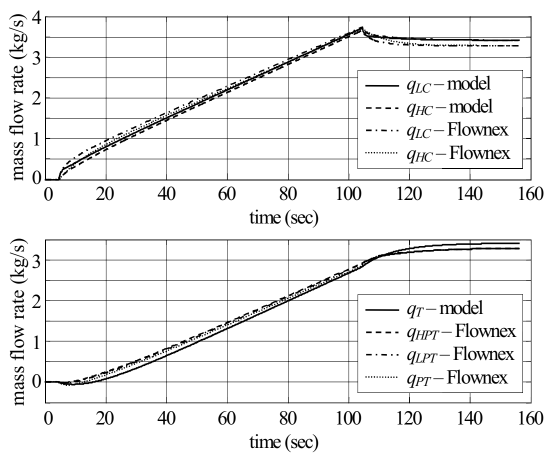

Figure 5. Perturbations in mass flow rate during helium injection.

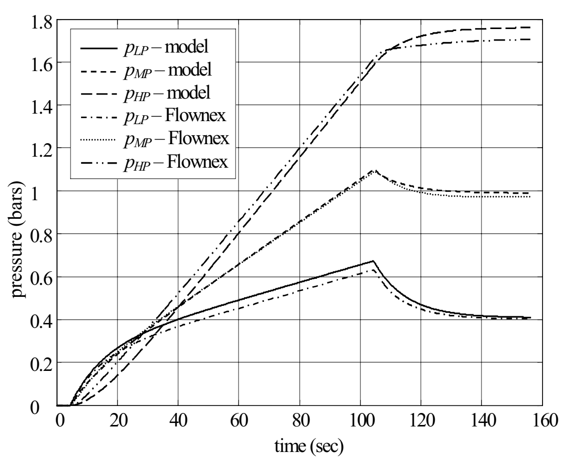

Figure 6. Perturbations in pressure during helium injection.

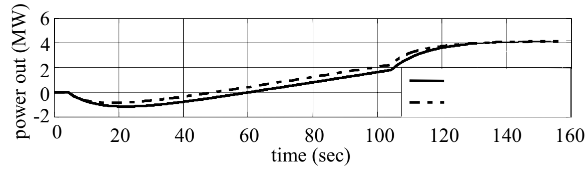

Figure 7. Perturbations in PT power output during helium injection.

In Figure 7 it can be seen that the power output of the power turbine decreases substantially before it starts to increase. This phenomenon is known as a non-minimum phase effect. By examining the state space equations, it can be deduced that it is caused by the fact that the injection of helium at the pre-cooler inlet (see Figure 1) causes a pressure rise at the PT outlet as well as at the LPC inlet which means that the pressure ratio of the compressor decreases. This has the immediate effect that the shaft speed starts to decrease (see Figure 4) causing the LPT pressure ratio to decrease and subsequently a drop in the PT inlet pressure. The drop in inlet pressure combined with the increase in outlet pressure of the PT causes a substantial decrease in the PT pressure ratio. Equation 10 shows that as the pressure ratio approaches unity, the difference between the inlet and outlet temperatures decreases, causing the drop in output power according to Equation 11.

The symbolic state space equations thus enable the visualization of the dominant parameters involved and an examination of the extent to which certain system variables are influenced in an occurrence such as helium injection at the pre-cooler inlet.

8. CONCLUSIONS AND RECOMMENDATIONS

The linear model developed is a simplified model that captures most of the dynamic behavior of the power conversion unit for small perturbations in the control valves. The model can be used in the design of the plant controllers and estimators, and can also be used as a system engineering tool. The results indicate that the linear model does in fact capture most of the dynamics when compared to Flownet simulations on a comparable network. The limitations of the model include:

- The model is not a true "open loop" model of the network, in that it assumes that flow rate control loops have already been closed around the control valves. That is, the model does not directly incorporate the effects of valve non-linearities, but simply assumes that a controller is already controlling the valves in such a way that a particular flow rate can be set to flow through any of the valves.
- The model does not include the effect of any dynamic changes in pressure drop between the turbo machines. As such, it assumes that the flow

resistance of all pipes in the network is zero. During development of the model, it was found that inclusion of flow resistance does influence the system dynamics to some extent, however for the purposes of keeping the model as simple as possible, these effects were left out.

- The linear model, as well as the Flownet model against which it was compared assumed constant temperatures at the reactor outlet and the inlets to the compressors. In reality, the reactor outlet temperature is controlled using the control rods and as such is subject to vary over time. For example, if the mass flow rate starts picking up, one would expect the reactor outlet temperature to start dropping slowly, until such time as the reactor controller can respond appropriately.
- Both the linear model and the Flownet model didn't allow for any dynamic changes in leak flows in the network. In reality, there will be dynamic changes in these flows, and this will have some effect on the dynamics. For the purposes of this model, these effects were considered to be small compared to the other dynamics.

The transfer functions for the model could not be derived in symbolic form due to the high order of the system. In order to gain a perspective on the factors that explicitly affect the transfer function poles and zeros, it would be valuable to perform sensitivity analyses on the transfer functions developed in the document. Such analyses will clearly highlight which parameters most strongly influence the transfer function poles and zeros.

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