LOW COMPLEXITY CONSTANT MODULUS BASED CYCLIC BLIND ADAPTIVE MULTIUSER DETECTION

J. B. Whitehead* and F. Takawira**

* Eskom, Resources & Strategy, CR&D, Private Bag 40175, Cleveland, 2022, South Africa
** School of Electrical, Electronic & Computer Engineering, University of KwaZulu-Natal, King George V Avenue, Glenwood, Durban, 4041, South Africa

Abstract: Periodically time varying (PTV) filters can offer significant performance gains over conventional filtering methods in DS-CDMA communications systems that are either multirate or corrupted by narrow band interference. This paper develops a new blind adaptive PTV linear multiuser detector based on the frequency shift (FRESH) architecture. The blind adaptive algorithm is based on the linearly constrained constant modulus algorithm (LCCMA), but is modified for the FRESH architecture. The resulting stochastic gradient algorithm has significantly less computational complexity than the recently proposed recursive and cyclic subspace tracking algorithms, and can offer comparable performance. The robustness of the FRESH-LCCMA algorithm is ensured through a proof of convergence which is carried out in the paper.

Keywords: Blind adaptive multiuser detection, constant modulus algorithm, cyclic algorithms, DS-CDMA, frequency shift filtering, multi-rate CDMA

1. INTRODUCTION

Future generation wireless communications systems based of DS-CDMA will be characterized by their flexibility to offer various services such as voice, data, and multimedia over the same air interface [1]. These heterogeneous services are characterized by their differing quality of service and data rate requirements. Multirate DS-CDMA has been suggested to provide such flexibility [1]. When multi-rate DS-CDMA is used in combination with short spreading codes, it is possible to exploit the cyclostationary statistics of the received signal vector to greatly enhance the reliability of the communications link. Cyclic multiuser detection (MUD) is one such technique that can be used to exploit the cyclostationary signal statistics and offers superior performance compared to conventional MUD techniques [2], [3]. Cyclic MUD also provides the capability to suppress co-channel narrow band interference (NBI) sources [4], [2] which may be present in overlay systems. This situation is typified if one considers operation in unregulated spectrum such as the industrial scientific and medical (ISM) band, which is occupied by both Bluetooth and wireless LAN standards like 802.11b.

Recently, the issue of the NBI suppression and MUD in multi-rate DS-CDMA has been addressed in [3], [2] where a blind cyclic recursive least squared (RLS) algorithm has been developed which is based on the minimum output energy (MOE) cost function of [5]. The convergence of this algorithm can be guaranteed, however it suffers from poor steady-state performance compared to (for example) the decision-directed (DD) mode of operation. Great lengths were made in [3] to reduce the computational complexity of this algorithm by making use of the block circulant structure of the associated matrices, however the high computational complexity may still prove to be inhibitive to implementation. Noting this point, the same authors developed a lower complexity cyclic MUD algorithm based on iterative cyclic subspace tracking in [6]. However, the authors acknowledge in [6] that even this algorithm may be too computationally complex for high dimension systems, and thus there is a need to develop low complexity, cyclic MUD.

This paper proposes the use of a frequency shift (FRESH) filter which is adapted via a modified version of the linearly constrained constant modulus algorithm (LCCMA) [7] to perform cyclic-MUD. This technique offers the flexibility of the FRESH architecture (as noted in [2], [4], [3], [6]) combined with the low complexity of a stochastic gradient algorithm for adaptation, which has a significantly lower computational complexity as compared with previously suggested cyclic algorithms in [3] and [6]. The LCCMA algorithm was chosen due to its proven convergence ability in DS-CDMA, and its superior steady-state performance as compared to the MOE cost function [8]. The robustness of this new algorithm is ensured as the proof of global convergence of FRESH-LCCMA is given in this paper.

The rest of the paper is outlined as follows: Section 2 introduces the multi-rate DS-CDMA system model, Section 3 outlines cyclic MUD as well as the new FRESH-
LCCMA algorithm proposed by this paper. The proof of
global convergence of this algorithm is given in Section
4, simulation results are presented in Section 5, and con-
cluding remarks are made in Section 6.

2. SYSTEM MODEL

The complex baseband representation of the received
signal of an asynchronous multirate DS-CDMA system at
time $t$ may be expressed by,

$$r(t) = \sum_{k=0}^{K-1} A_k \alpha_i \sum_{i=0}^{N-1} b_k (i) s_i (t - \tau_k - iT_{s,i}) + n(t) \quad (1)$$

where $K$ is the number of users, $A_i$ is the amplitude of
the $k$th user, $\alpha_i$ accounts for the complex gain of the channel,
and $b_k (i), \pm 1$ is the $k$th user’s $i$th transmitted data symbol.
The unit energy signature waveform of the $k$th user is denoted
by $s_k (t)$, $\tau_k$ is the time offset, and $T_{s,i}$ is the bit
duration of the $k$th user. The additive white Gaussian noise (AWGN)
term $n(t)$ has power spectral density $\sigma_i^2$.

A variable spread length (VSL) dual-rate access scheme is considered
in this paper to illustrate the effectiveness of the new cyclic-algorithm.
In such a scheme the chip duration $T_c$ for both data rates is constant but the number
of chips per code differs between the high rate and low rate users, hence the VSL designation.
In such a system the high rate user transmit at a rate $L$ faster than the low rate users, and are thus assigned spreading sequences that are shorter by a factor $1/L$ to ensure that spread signals of both rates occupy the same amount of
bandwidth. The signature waveforms are given by,

$$s_k^{HR} (t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c_k^{HR} (n) \text{rect}_T (t - nT_c)$$

$$s_k^{LR} (t) = \frac{1}{\sqrt{LN}} \sum_{n=0}^{LN-1} c_k^{LR} (n) \text{rect}_T (t - nT_c) \quad (2)$$

where $HR$ and $LR$ specify high rate and low rate respectively, $N$ is the number of chips in the HR spreading sequence, $\{ c_k^{HR} (n) \}$ is the spreading code assigned to user $k$, and $\text{rect}_T (t)$ is the unit-height rectangular pulse support on $[0,T_c]$. From here on let the desired user be user
1, and be assigned a HR spreading code.

The receiver samples the output of a chip-matched filter
to convert the received signal into discrete samples.

The samples within processing window $i$ are stacked to form a vector $r (i)$, which is used to detect $b_1 (i)$, which is the $i$th bit of the desired user. This paper considers the case where processing window $i$ spans bit interval $i$ of the desired user, and the detector operates on a symbol-by-symbol basis. For the sake of brevity only, this paper considers the case of rectangular shaped chip pulses and a sample rate equal to that of the chip rate.

3. CYCLIC LINEAR MULTIUSER DETECTION

Consider the problem of detecting desired user 1 (a high rate user), then the well known linear MMSE filter is given by,

$$w = C^{-1} p \quad (3)$$

$$C = E \{ r (i) r (i)^T \}$$

$$p = E \{ h r (i) \}.$$

The spreading waveforms of the low-rate users are periodically time varying (PTV) in the processing window, and this causes the sequence of covariance matrices conditioned on temporal index $l$ to also be PTV,

$$C(l) = E \{ r (l) r (l)^T \}$$

$$= C_{LR} + C_{LR} (l) + \sigma_i^2 I \quad (4)$$

since $C_{LR} (l)$, the covariance matrix of the low-rate MAI, is periodic in $l$ with period $L$. The unconditioned MMSE receiver defined in (3) is thus not optimal for any particular bit epoch and would suffer worse performance compared to the PTV filter defined by,

$$w (l) = C^{-1} (l) p. \quad (5)$$

The implementation of PTV filters is a mature topic [9]. The simplest implementation of which is the filter bank where $L$ filters in parallel are employed and are selected and sampled sequentially. Another implementation which has attracted attention for cyclic MUD is the FRESH structure [9]. The FRESH filter makes use of the Fourier series representation of the periodic sequence of the filter coefficients $\{ w (l) \}$, and by doing so confines the time varying component of the PTV-filter to a bank of complex oscillators, see Fig. 1.
Define the stacked vector of \( L \) frequency shifted versions of the received vector of samples as,
\[
\mathbf{\tilde{r}}(i) = \left[ r^T(i), r^T(i) e^{\left(\frac{j2\pi}{L}\right)}, \ldots, r^T(i) e^{\left(\frac{j2\pi(L-1)}{L}\right)} \right]^T
\]
then the stacked vector \( \tilde{\mathbf{w}} = [w_{0}, w_{1}, \ldots, w_{L-1}]^T \) of FRESH filter coefficients (which is time invariant) is given by,
\[
\tilde{\mathbf{w}} = \mathbf{C}^{-1} \tilde{\mathbf{p}}
\]
where,
\[
\tilde{\mathbf{p}} = E \{ b_i \mathbf{r}(i) \}
\]
\[
\mathbf{\hat{C}} = E \{ \mathbf{r}(i) \mathbf{r}^H(i) \}
\]
\[
= \begin{bmatrix}
\mathbf{C}^{(0)} & \mathbf{C}^{(1)} & \ldots & \mathbf{C}^{(M-1)} \\
\mathbf{C}^{(M-1)} & \mathbf{C}^{(0)} & \ldots & \mathbf{C}^{(M-2)} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{C}^{(L-1)} & \mathbf{C}^{(L-2)} & \ldots & \mathbf{C}^{(0)}
\end{bmatrix}
\]
and \( \mathbf{C}^{(m)} = E \{ \mathbf{r}(i) \mathbf{r}^H(i) e^{j2\pi m/L} \} \), which is the \( m \)th matrix of Fourier coefficients of the Fourier transform of the sequence of matrices \( \{ \mathbf{C}(i) \} \).

3.1 Cyclic Algorithm

The FRESH-LCCMA directly operates on the stacked frequency shifted vector \( \mathbf{\tilde{r}}(i) \), and thus jointly optimizes each filter bank corresponding to the different frequency shifts. In so doing the stacked vector of FRESH filter coefficients \( \tilde{\mathbf{w}} \) is treated as if it were a single filter. FRESH-LCCMA makes use of its knowledge of the desired user’s spreading sequence to linearly constrain the adaptive algorithm to ensure the desired user is captured by the algorithm, and not any other constant modulus signal which may be present. Based on the CMA2-2 [10] cost function, the FRESH-LCCMA cost function is given by,
\[
\min_{\mathbf{s}^{'}, \mathbf{s} = 1} J(\tilde{\mathbf{w}}(i)) = E \left\{ (y(i)^2 - R_z)^2 \right\}
\]
where
\[
\mathbf{s}^{'}, \mathbf{0}_{L-1} = \text{vector of length } N, \quad \text{and} \quad y(i) = \tilde{\mathbf{w}}^H(i) \mathbf{r}(i).
\]
The update step is
\[
\tilde{\mathbf{w}}_z(i+1) = \tilde{\mathbf{w}}_z(i+1) + \mu y(i) F_z(i) \mathbf{r}_z(i)
\]
where
\[
F_z(i) = y(i) \left( y(i)^2 - R_z \right),
\]
\[
\tilde{\mathbf{w}}(i) = \mathbf{s}_z + \tilde{\mathbf{w}}_z(i),
\]
and \( \mathbf{r}_z(i) = \mathbf{B} \mathbf{r}(i), \quad \mathbf{B} = \mathbf{I} - \mathbf{s}_z^H \mathbf{s}_z^H \), such that \( \tilde{\mathbf{r}}_z(i)^H \mathbf{s}_z^{'}, \mathbf{s}_z^{''} = 0 \),

### TABLE I

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCCMA</td>
<td>( O(NM) )</td>
</tr>
<tr>
<td>MOE-RLS</td>
<td>( O\left(\frac{NM^2}{L}\right))</td>
</tr>
<tr>
<td>PAST(d)</td>
<td>( O\left(\frac{NM}{L}\right))</td>
</tr>
<tr>
<td>FRESH-LCCMA</td>
<td>( O\left(\frac{LMN}{L}\right))</td>
</tr>
<tr>
<td>Cyclic MOE-RLS</td>
<td>( O\left(\frac{L(NM^2)}{L}\right))</td>
</tr>
<tr>
<td>Cyclic PAST(d)</td>
<td>( O\left(\frac{NL^2}{L}\right))</td>
</tr>
</tbody>
</table>

Where: \( r \) - dimension of signal subspace, \( N \) - number of chips in processing window, \( M \) - samples per chip, \( L \) - period of system.

Source: [3], [6].

which ensures that \( \tilde{\mathbf{w}}_z \) remains orthogonal to \( \mathbf{s}_z \), which in turn ensures the linear constraint \( \tilde{\mathbf{w}}^H \mathbf{s}_z^{'}, \mathbf{s}_z^{''} = 1 \) is always met.

### 3.2 Comments on Complexity

The relative complexity of various blind adaptive MUD schemes is presented in Table 1. These algorithms are representative of three categories of blind adaptive MUD algorithm: stochastic gradient (LCCMA [7]), recursive least squares (MOE-RLS [3]), and subspace tracking (PAST\(d\) [6]). Cyclic MOE-RLS scales linearly in complexity with the periodicity of the system (which is desirable), but it can be seen that the overall complexity is far higher than FRESH-LCCMA. Reference [6] argues that usually \( P < \left(\frac{NM}{L}\right) \), and therefore the cyclic PAST\(d\) algorithm has a lower computational complexity than the cyclic MOE-RLS algorithm. It can be seen that the complexity of FRESH-LCCMA is lower than cyclic PAST\(d\), and that as the periodicity of the system increases so the relative complexity of cyclic PAST\(d\) increases over FRESH LCCMA.

### 4. CONVERGENCE ANALYSIS

Convergence is proved for the case where there is no AWGN, as is customary [11], [7]. Firstly, a unique stationary point is shown to exist using the first derivative of the cost function with respect to the filter coefficients. This stationary point is then shown to coincide with the decorrelating detector. On inspection of the associated Hessian matrix, it is shown that this stationary point on the cost surface is a global minimum, which completes the analysis. The minimum condition required for convergence is then given.

The signal associated with user \( k \) at the input to FRESH filter \( \tilde{\mathbf{w}}(i) \) is given by \( A_b_k \mathbf{s}_k(i) \mathbf{h}_k(i) \), where
\[
\mathbf{s}_k(i) = \begin{bmatrix} s_k^1(i), \ldots, s_k^L(i) e^{\left(\frac{j2\pi k}{L}\right)} \end{bmatrix}^T.
\]
In a similar fashion to [11] and [7], define the (PTV) signal from user \( k \) at the output of the filter as,
\[ u_k(i) = A_k \mathbf{w}^H \mathbf{s}_k(i). \] (9)

This sequence only takes on \( L \) distinct values, denote this sequence by \( \{u_k(1), \ldots, u_k(L)\} \). By making use of the property of the periodic sequence:

\[ E \left[ (u_k(i))^2 \right] = \frac{1}{L} \sum_{i=1}^{L} (u_k(i))^2, \] (10)

the expanded cost function (7),

\[
\min_{\mathbf{w}^H \mathbf{s} \in \mathbb{C}^n} J \left( \mathbf{w}(i) \right) = E \left[ \{y(i)\}^2 - R_1 \right] \\
= |y(i)|^2 - 2 |y(i)| R_1 + R_1^2 \\
= |\mathbf{w}^H(i) \mathbf{f}(i)|^2 - 2 |\mathbf{w}^H(i) \mathbf{f}(i)| R_1 + R_1^2,
\]

may be expressed in terms of \( u_k(i) \) using the terms,

\[
E \left[ |\mathbf{w}^H(i) \mathbf{f}(i)|^2 \right] = E \left[ \sum_{k=1}^{K} A_k b_k(i) u_k(i) \right]^2 \\
= E \left[ \sum_{k=1}^{K} b_k(i) u_k(i) \right]^2 \\
= E \left[ \sum_{k=1}^{K} |u_k(i)|^2 \right] \\
= \frac{1}{L} \sum_{k=1}^{K} u_k(i) u_k(i)^* \\
= \frac{1}{L} \sum_{k=1}^{K} \sum_{n=1}^{N} u_i(i) u_i(i)^* \\
\]

Define the vector,

\[
\mathbf{u} = [u^T_1, \ldots, u^T_K]^T
\]

then the cost function \( \mathbf{E}\mathbf{u}^T \mathbf{u} \) may be expanded into the terms,

\[
E \left[ \mathbf{u}^T \mathbf{u} \right] = \frac{1}{L} \sum_{k=1}^{K} \sum_{i=1}^{L} |u_k(i)|^2 \\
+ \frac{6}{L} \sum_{k=1}^{K} \sum_{n=1}^{N} u_i(i) u_i(i)^* \\
= \frac{1}{L} \sum_{k=1}^{K} \sum_{i=1}^{L} \left| u_k(i) u_k(i)^* \right|^2 \\
+ \frac{6}{L} \sum_{k=1}^{K} \sum_{n=1}^{N} u_i(i) u_i(i)^* \\
- 2R_1 \frac{1}{L} \sum_{k=1}^{K} \sum_{i=1}^{L} u_k(i) u_k(i)^* + R_1^2.
\]

**Proposition:** The linear constraint \( \mathbf{w}^H \mathbf{s} = \mathbf{1} \) ensures

\[
\|u_k(i)\| = A_k.
\]

**Proof:**

\[
\|u_k(i)\| = A_k \left[ \mathbf{w}^H(i) \mathbf{s}_k(i) \right] \\
= A_k \left[ \mathbf{w}^H(i) (\mathbf{s} + \mathbf{s}_k(i)) \right] \\
\]

where \( \mathbf{s} = [s^T_1, \ldots, s^T_{L-N}]^T \) and \( \mathbf{s}_k(i) \) is defined as

\[
\mathbf{s}_k(i) = [0^T, s^T_1 e^{2\pi j i}, \ldots, s^T_{L-N} e^{2\pi j (L-1-i)L}]^T
\]

then \( \mathbf{s}^T \cdot \mathbf{s}_k(i) \), and therefore,

\[
\|u_k(i)\| = A_k \left[ \mathbf{w}^H(i) \mathbf{s}_k(i) \right] \\
= A_k + \|\delta(l)\|^2 \\
= A_k
\]

where \( \|\delta(l)\| = A_k \mathbf{w}^H(i) \mathbf{s}_k(i) \), and therefore

\[
\|u_k(i)\| = A_k
\]

The \( u_k(i) \) terms are complex and therefore the first partial derivative of the cost function is given with respect to the real and imaginary components of \( u_k(i) \), where \( u_k(i) = x_k(i) + jy_k(i) \), using the block vector notation,

\[
\mathbf{\phi}(\mathbf{u}) = \begin{bmatrix} \mathbf{\phi}(\mathbf{u}) \\ \mathbf{x} \\ \mathbf{y} \end{bmatrix}
\]

where \( \mathbf{x} = [x^T_1, \ldots, x^T_K]^T \) and \( \mathbf{y} = [y^T_1, \ldots, y^T_K]^T \), and \( \mathbf{y} \) is similarly defined. The above expression can then be further expanded into the terms,

\[
\mathbf{\phi}(\mathbf{u}) = \begin{bmatrix} \mathbf{\phi}(\mathbf{u}) \\ \mathbf{x} \end{bmatrix}
\]

where,

\[
\mathbf{\phi}(\mathbf{u}) = \begin{bmatrix} \mathbf{\phi}(\mathbf{u}) \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{\phi}(\mathbf{u}) \\ \mathbf{x}_K \end{bmatrix}
\]

and \( \mathbf{\phi}(\mathbf{u})^T \mathbf{y} \), \( \mathbf{\phi}(\mathbf{u})^T \mathbf{y}_k \) are similarly defined. The partial derivatives that constitute (19) are in turn given by,
where
\[
\frac{\partial^2 \phi(u)}{\partial^2 x} = \begin{bmatrix}
\frac{\partial^2 \phi(u)}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 \phi(u)}{\partial x_1 \partial x_k} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 \phi(u)}{\partial x_k \partial x_1} & \cdots & \frac{\partial^2 \phi(u)}{\partial x_k \partial x_k}
\end{bmatrix},
\]

and similarly,
\[
\frac{\partial^2 \phi(u)}{\partial y} = \begin{bmatrix}
\frac{\partial^2 \phi(u)}{\partial y_1 \partial y_1} & \cdots & \frac{\partial^2 \phi(u)}{\partial y_1 \partial y_l} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 \phi(u)}{\partial y_l \partial y_1} & \cdots & \frac{\partial^2 \phi(u)}{\partial y_l \partial y_l}
\end{bmatrix},
\]

and \(\frac{\partial^2 \phi(u)}{\partial y_1 \partial y_k}\) and \(\frac{\partial^2 \phi(u)}{\partial y_l \partial y_l}\) are similarly defined. Then
\[
\frac{\partial^2 \phi(u)}{\partial x_k} = \begin{bmatrix}
\frac{\partial^2 \phi(u)}{\partial x_k x_1} & \cdots & \frac{\partial^2 \phi(u)}{\partial x_k x_k} \\
\frac{\partial^2 \phi(u)}{\partial x_k y_1} & \cdots & \frac{\partial^2 \phi(u)}{\partial x_k y_l}
\end{bmatrix},
\]

and the other sub-matrices associated with \(\frac{\partial^2 \phi(u)}{\partial y_k \partial y_k}\) and \(\frac{\partial^2 \phi(u)}{\partial y_l \partial y_l}\) are similarly defined. The elements of the Hessian matrix are then given by,
\[
\frac{\partial^2 \phi(u)}{\partial x_k (l) \partial x_{k'} (l')} = \begin{cases}
\frac{4}{L} \left( 3 x_1^2 (l) + y_1^2 (l) \right) + 3 \sum_{n=1}^{K} \left( x_n^2 (l) + y_n^2 (l) \right) - R_s & \text{for } k = k', l' = l \\
\frac{24}{L} x_1 (l), & \text{for } 24 x_1 (l), k' \neq k, l' = l \\
0, & \text{otherwise}.
\end{cases}
\]
The FRESH-LCCMA cost function is

\[
\frac{\partial \phi(i)}{\partial y_l(l)} y_\ell(l) = \begin{cases} 
\frac{4}{L} \left( x_l^2(l) + 3 y_l^2(l) + 3 \sum_{i=1}^{k} (x_i^2(l) + y_i^2(l)) - R_z \right), & \text{for } k = k, l' = l \\
\frac{24}{L} y_l(l), & k' \neq k, l' = l \\
0, & \text{otherwise.}
\end{cases}
\]  

From (27)-(30), it can be seen that the off-diagonal elements of the Hessian matrix at the stationary point are zero. The main diagonal of the Hessian matrix is fully determined by \( \frac{\partial \phi(u)}{\partial x_i^2} x_i^2(l), \frac{\partial \phi(u)}{\partial y_i^2} y_i^2(l), \frac{\partial \phi(u)}{\partial x_i^2} x_i^2(l), \) and \( \frac{\partial \phi(u)}{\partial y_i^2} y_i^2(l). \) These terms are in turn guaranteed to be positive if the following corresponding constraints are met,

\[
3u_i(l) u_i(l) - R_z \geq 0 \quad (31)
\]

\[
3u_i(l) u_i(l) - R_z \geq 0 \quad (32)
\]

\[
3x_i^2(l) + y_i^2(l) - R_z \geq 0 \quad (33)
\]

\[
x_i^2(l) + 3y_i^2(l) - R_z \geq 0. \quad (34)
\]

Constraints (31) and (32) are the same and are automatically satisfied when the stricter constraints (33) and (34) are met. It has already been proved that \( x_i^2(l) + y_i^2(l) \geq A_i^2, \) and therefore a sufficient condition to ensure constraints (33) and (34) are satisfied is

\[
A_i^2 - R_z \geq 0 \quad (35)
\]

thus ensuring that the Hessian matrix is positive definite. Under this condition the stationary point is a minimum point on the cost surface, thus completing the convergence proof of the FRESH-LCCMA algorithm.

It is worth noting at this point that \( x_i^2(l) \gg y_i^2(l), \) since the vast majority of the energy at the output of the filter associated with the desired user lies in the real domain as BPSK signalling and phase synchronisation is assumed. This leads to the conclusion that \( x_i^2(l) + 3y_i^2(l) \equiv A_i^2, \) and therefore the constraint imposed in (35) is not overly loose, which indicates that the constraint for convergence of the FRESH-LCCMA is in fact more sensitive to \( A_i^2 \) overestimation than the LCCMA is, where it was proved in [11] that a sufficient condition for LCCMA convergence is \( 3A_i^2 - R_z \geq 0 \).

The same conjectures made in [7] with regard to the convergence of the LCCMA in an AWGN channel apply in this case, and it is thus assumed that by properly selecting the value of \( R_z \) the FRESH-LCCMA cost function is strictly convex in an AWGN channel.

5. RESULTS

Simulation examples are presented in this section to illustate the effectiveness of the new algorithm. The VSL multi-rate access scheme was implemented using orthogonal Gold codes of length 16 and 64 for the high-rate and low-rate users respectively. Orthogonal Gold codes may be constructed from conventional Gold codes (length 15 and 63 respectively) by adding a “-1” to the end of each conventional Gold code. This allowed the LR code length to be an integer multiple of the HR code. The signal to noise ratio (SNR) was set to 15dB’s. The MAI ratio is defined as \( A_i/A_j, \) where all the interfering users transmit at the same amplitude. There are 3 low rate users and 3 high rate users present in all simulations. A synchronous DS-CDMA system is considered in order to reduce the simulation time.

The output signal to interference plus noise ratio (SINR) at each time step is employed as a metric to quantify the performance of the adaptive algorithms. The SINR is an important metric as it enables an accurate estimate of the bit error rate (BER) [12] or quality of service that a network subscriber experiences. It may also be used to drive power-control algorithms. The output SINR at bit epoch \( i \) of the FRESH filter may be defined as:

\[
SINR_{\text{FRESH}}(i) = \frac{\left( \sum_{i=1}^{L} w(i) S \tilde{w}(i) \right)^2}{\sum_{i=0}^{L} w(i) S \left( C - A_i^2 S \right) \tilde{w}(i)}. \quad (36)
\]

This SINR value is equivalent to the mean SINR over one complete period of the system, and is a more useful metric to quantify the performance of a PTV filter. This is because the output SINR at each bit epoch of a conventional PTV filter is cyclic in nature, and thus plots of the SINR versus time would yield a broad area instead of a line, which is unsatisfactory. A length \( L \) sliding window is thus used to define the SINR of the conventional PTV filter at bit epoch \( i \) to enable a direct comparison between the two realizations of the PTV filter, as the optimum SINR defined by (36) and (37) are the same.

The convergence dynamics of the new FRESH-LCCMA algorithm from single user matched filter to the MMSE filter are presented in Figs. 2 (a)–(c) where each SINR curve is the ensemble average of 100 independent simulation runs. The sequence of figures is intended to illustrate the relative performance of the new FRESH-LCCMA algorithm and the recently proposed cyclic-RLS algorithm [3], the cyclic subspace tracking algorithm [6], and the conventional non-cyclic LCCMA [7] as a function of MAI level. The sequence of figures thus models an increasing level of severity of the near-far problem which is one of the fundamental problems or limitations of DS-CDMA systems, and which signal processing schemes such as MUD hope to circumvent. The MAI ratio is 0dB, 3dB, and 10dB for Figs. 2 (a), (b), and (c) respectively.
Fig. 2. Convergence dynamics and steady-state behavior of the mean SINR (over a sliding window) of cyclic and non-cyclic MUD’s with an increasing near-far problem: (a) 0dB, (b) 3dB, and (c) 10dB MAI ratio. There are 6 users present (3 HR and 3 LR) in all the systems and the SNR of the desired user is 15dB.
The cyclic MOE-RLS forget factor was set to 0.9995, and the cyclic PASTd forget factor was set to 0.99 for Figs. 2 (a)–(c). The step sizes for the other adaptive algorithms, given in sequence for Figs. 2 (a)–(c) are, FRESH-LCCMA: 5E-3, 1E-3, 2E-5, LCCMA: 1E-2, 5E-3, 1E-5.

In those figures the advantage that cyclic MUD has over non-cyclic MUD is apparent as the optimal (MMSE) SINR gain that cyclic MUD has over conventional linear MUD is 2.62dB, 5.47dB, and 9.29dB for Figs. 2 (a), (b), and (c) respectively. This inherent advantage is evident when comparing the relative performance of FRESH-LCCMA and LCCMA in Figs. 2 (a)–(c), where the superior performance (in terms of both convergence speed and steady-state SINR) is evident.

The performance of FRESH-LCCMA is seen to compare favorably with existing cyclic MUD schemes. The cyclic MOE-RLS has the advantage of a rapid convergence rate due to the use of the RLS update scheme, however the MOE cost function causes poor steady-state performance as compared to FRESH-LCCMA. As the near-far problem increases the FRESH-LCCMA algorithm requires a smaller step size to ensure stability of the algorithm, and this in turn reduces its convergence speed. The cyclic MOE-RLS algorithm does not suffer from this effect; however its worse steady-state performance is clearly evident in Fig. 2 (c). The cyclic subspace tracking algorithm suffers from poorer convergence speed as the near-far ratio increases, as was found in [6]. FRESH-LCCMA appears to have superior performance to the cyclic subspace tracking algorithm for the given scenarios.

6. CONCLUSION

In this paper, the problem of low complexity, robust, fast converging blind adaptive cyclic MUD has been dealt with by proposing the use of a modified LCCMA algorithm on the FRESH receiver structure. The proof of convergence of the adaptive algorithm was made possible by defining a mean output energy term per interfering user and showing that it tends to zero at convergence. It was found that the FRESH filter based on the LCCMA provides a viable alternative to existing schemes, at a substantially lower computational complexity.

7. REFERENCES