IDENTITY CONFIDENCE ESTIMATION OF MANOEUVRING AIRCRAFT

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Abstract: A radar system observes an aircraft once during each scan of the airspace, and uses these observations to construct a track representing a possible route of the aircraft. However when aircraft interact closely there is the possibility of confusing the identities of the tracks. In this study multiple hypothesis techniques are applied to extract an identity confidence from a track, given a set of possible tracks and observations. The system utilises numerous estimation filters internally and these are investigated and compared in detail. The Identity Confidence algorithm is tested using a developed radar simulation system, and evaluated sucessfully against a series of benchmark tests.

Key Words: radar, tracking, identity, confidence

1. INTRODUCTION

Radar operates in a noisy world. It is the task of radar tracking software to keep track of an airplane, given noisy measurements and aircraft estimates. This is difficult since the measurements from different airplanes can be mixed and *false detections* are also a possibility. Sometimes the target remains undetected for undetermined lengths of time causing *missed detections.*

It is therefore unrealistic to expect a tracker to operate without error indefinitely, and identity checking mechanisms are used to ascertain that the aircraft that enters the airspace is in fact the aircraft that lands. In aerospace, the airplane identity is usually confirmed with the use of transponders or by means of radio communication. In military situations however, such aircraft identification is not always possible.

It is sometimes unavoidable for airplanes to manoeuvre close to one another, causing situations where identity confirmation is not straightforward. In combat situations radio silence is often enforced, and visual inspection often involves close quarters flight patterns.

Therefore in case of non-operational transponders, it is useful to determine to what extent two closely encountering aircraft might be confused with one another. Figure 1 illustrates a scenario involving two confirmed airplanes A and B with end positions supplied by a tracker. The tracker decided in this case that plane A moved to position A and plane B to position B, but the situation might have been indeed the opposite.

For a radar operator looking at aircraft interacting on the screen it can be useful to have an analysis tool that shows the identity confidence probability for a specific track. This can offer useful supplemental advice to aid human judgement during difficult decision making.

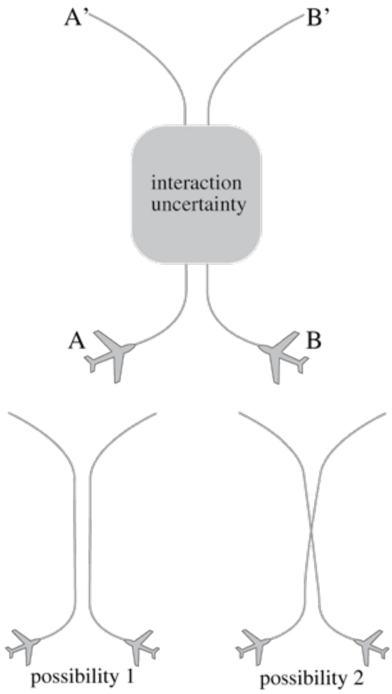


Figure 1: The interaction of two flight paths

The following factors play a role in this problem:

- **History**: The movement history of an airplane is important, and can be used to predict future positions. Past behaviour is also a good indication of future behaviour, for example an airplane starting a manoeuvre is more likely to be unpredictable than one flying a straight path.
- **Dynamics**: When engaged in an manoeuvre, aircraft dynamics can restrict the kind of motion that is achiev-

able. A pilot can make a $5g$ turn in combat, but anything higher is likely to be to his detriment.

- **Radar characteristics**: The radar site location and noise parameters largely determine the extent of the confusion. A radar typical makes greater azimuthal error than range error.

Approaching the problem from the radar tracking side, many of these aforementioned factors can be easily integrated. Kalman filters for example are designed to cope with noise corrupted measurements.

Radar tracking is a field with many methodologies. The simplest is the Global Nearest Neighbour (GNN) technique that *gates* (i.e. selects) measurements around each track, and then associates each measurement with a track to minimize the sum of measurement-to-track distances.

Bar Shalom [1] is a proponent of Probabilistic Data Association (PDA), where every track is updated by a weighted sum of all observations within the gating distance. Special attention needs to be paid to creation of new tracks and track interaction.

Reid [2] introduced the Multiple Hypothesis Tracker (MHT) that operates by considering every possibility of data association, and assigning a probability to each hypothesis. Instead of making a hard decision like the other techniques, the possibilities are propagated into the future with the idea that future data will resolve uncertainties. In a MHT hypothesis an estimation filter is assigned to each aircraft to give the best possible estimate of position and velocity.

After first covering general radar background, we will investigate estimation filters in the **Track Modeling** section after which it will be integrated with the MHT in the **Track Management** section. Multiple Hypothesis techniques seem promising to handle the desired factors, and we extend it from a normal tracker to serve as an analysis system.

2. RADAR BACKGROUND

The radar system of this study is a mechanically scanned search radar, with a rotating antenna that covers the entire search volume after one rotation. Observations (also known as *hits*) are received at regular intervals (typically 4 - 10 seconds), and from this a *tracker* creates *tracks* that represent estimated aircraft motion. A functioning radar device of a local company is used as subject for further simulations, and this radar has a search volume range of 65 km to a height of 8 km. It cannot make height detections, so only azimuthal and range measurements are therefore available.

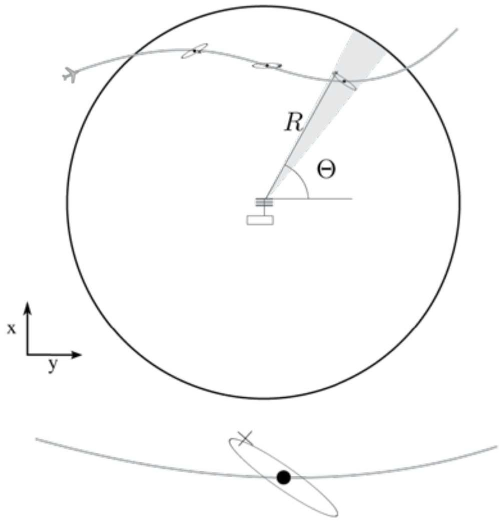


Figure 2: View from top of a radar situation, with detail of a noise covariance.

The radar observations are corrupted by noise as depicted in Figure 2, and the noise parameters are specified properties of a radar system (for our system, azimuth deviation 0.01222 radians and range deviation 18m). Visual sightings and position communication via radio can augment a tracking system. In combat situations Identification Friend or Foe (IFF) systems are used to identify aircraft, but usually only over specified zones. Radio silence during combat is however the standard. Thus even if an aircraft is identified at a specific moment, that certainty could be lost during close encounters with other targets.

3. TRACK MODELING

The radar observations received could be used as the approximated position of an aircraft, but there is more information available than this noise-corrupted data leading to better estimates. Two methods of tracking are discussed: Kalman filter and the Integrated Multiple Model technique.

3.1. *Kalman filter*

The Kalman filter [3] is a recursive filter with its gain being continuously adjusted based on the measurements received, the target dynamics and the noise models. The Kalman gain determines to what extent the estimate is either influenced by the measurement or influenced by the dynamic process model.

We used a linear Cartesian filter, with 4 or 6 states depending on whether acceleration is included as part of the model. A 4-state filter will perform better on simple linear motion, while a 6-state will track a turn better.

Predict:

The prediction for the state at time k is made before the measurement is received, by multiplying the state transition matrix A with the previous estimate. The covariance of the estimated state error P is predicted in a similar way, with the process noise covariance Q_k taking into account inaccuracies of the dynamic model

$$\begin{aligned} \hat{x}(k \mid k-1) &= A\hat{x}(k-1 \mid k-1) \\ P(k \mid k-1) &= AP(k-1 \mid k-1)A^T + Q_k. \end{aligned} \quad (1)$$

The choice of Q covariance is an important matter since it expresses the dynamics of the aircraft. A Kalman filter with a large covariance will track difficult manoeuvres better, but with estimation performance dropping.

Update:

The innovation ϵ is the difference between predicted measurement and the actual measurement

$$\epsilon(k) = z(k) - Hx(k \mid k-1). \quad (2)$$

The innovation covariance $S(k)$ of the estimated measurement includes the measurement noise covariance R_k, and this is used in the Kalman gain $K(k)$

$$S(k) = HP(k \mid k-1)H^T + R_k \quad (3)$$
$$K(k) = P(k \mid k-1)H^T S(k)^{-1}. \quad (4)$$

Now the state estimate $\hat{x}(k)$ is calculated by taking the state prediction and adjusting it according to the innovation and the Kalman gain. The state error covariance estimate is calculated for time step k with the use of the Kalman gain

$$\begin{aligned} \hat{x}(k) &= \hat{x}(k \mid k-1) + K(k)\epsilon(k) \\ P(k \mid k) &= (I - K(k)H)P(k \mid k-1). \end{aligned} \quad (5)$$

3.2. *Interacting Multiple Model filter*

The Interacting Multiple Model (IMM) estimator (as described by [4, p 455]) mixes the estimates from r Kalman filters according to how well it tracks the object. A Markov model describes the transition between the filter modes, meaning that there are specific probabilities that a target will change from one manoeuvre configuration to another.

In this way a low manoeuvre Kalman filter can be used for straight sections, while a high manoeuvre Kalman filter can be used for sections with sudden direction changes. As one performs better, its influence is increased in the mixed output, and similarly decreased as the performance drops.

Calculation of mix probabilities:

This step calculates the probability that one mode switched to another. The variable $\mu_{i|j}$ expresses the probability that mode M_i was active at $k-1$ given that M_j is active at k

$$\mu_{i\ |j}(k-1 \mid k-1) = \frac{p_{ij}\mu_i(k-1)}{\sum_{i=1}^{r} p_{ij}\mu_i(k-1)} = \frac{p_{ij}\mu_i(k-1)}{\bar{c}_j}. \quad (6)$$

Mixing:

Each filter calculates a new state by mixing all the filters together according to the mode transition probabilities, where r is the number of modes and $j = 1 \ldots r$

$$\begin{aligned} \hat{x}^{0j}(k-1 \mid k-1) &= \sum_{i=1}^{r} \hat{x}^i(k-1 \mid k-1) \\ &\quad \mu_{i|j}(k-1 \mid k-1). \end{aligned} \quad (7)$$

The covariance is combined in a corresponding manner

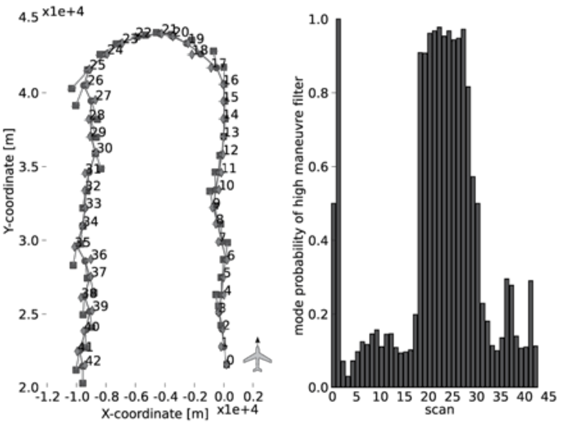
$$\begin{aligned} & P^{0j}(k-1 \mid k-1) \\ = & \sum_{i=1}^{r} \mu_{i|j}(k-1 \mid k-1)\Big\{P(k-1 \mid k-1) + \\ & \left[\hat{x}^i(k-1 \mid k-1) - \hat{x}^{0j}(k-1 \mid k-1)\right] \cdot \\ & \left[\hat{x}^i(k-1 \mid k-1) - \hat{x}^{0j}(k-1 \mid k-1)\right]'\Big\}. \end{aligned} \quad (8)$$

Filter update and mode probability calculation:

With $\hat{x}^{0j}$ and P^{0j} assigned as mixed states of filter j, measurement $z(k)$ now updates each individual filter estimates in using these mixed states.

The likelihood Λ_j associated with the filter j is calculated with use of the innovation covariance S^{0j}, and assumed to be Gaussian with mean at the state position estimate $\hat{z}^{0j}$. Each filter has a mode probability μ, that represents the probability that the current filter is active given the measurement history. With $\bar{c}_j$ given by Equation 6, the likelihood and probability are given by

$$\begin{aligned} \Lambda_j(k) &= N[z(k); \hat{z}^{0j}, S^{0j}] \\ \mu_j(k) &= \frac{\Lambda_j(k)\bar{c}_j}{\sum_{j=1}^{r} \Lambda_j(k)\bar{c}_j}. \end{aligned} \quad (9)$$



(a) (b)

Figure 3: a) IMM tracking a $2g$ turn.
b) Mode probability of the high manoeuvre filter.

The mode probability for time k is calculated using the likelihood derived during the update step, together with the Markov transition probabilities and the mode probabilities from time $k-1$.

Estimate and covariance output:

Up to now each filter is mixed separately, and only influences each other during the mixing state. An output can be determined at any time by mixing these filters using the mode probabilities as weights. So it is similar to the mixing step in Equations 7 and 8, but this output is not fed back into the algorithmic loop

$$\hat{x}^j(k \mid k) = \sum_{j=1}^{r} \hat{x}^j(k \mid k)\mu_j(k \mid k) \tag{10}$$

$$\begin{aligned} P(k \mid k) \quad = \quad & \sum_{j=1}^{r} \mu_j(k) \Big\{ P^j(k \mid k) + \\ & \left[\hat{x}^j(k \mid k) - \hat{x}(k \mid k)\right] \cdot \\ & \left[\hat{x}^j(k \mid k) - \hat{x}(k \mid k)\right]' \Big\}. \end{aligned} \tag{11}$$

IMM Example:

Two filters form part of the ensemble: a 4-state low process noise filter to handle straight predictable path sections and a high process noise 6-state filter for tracking more intensive manoeuvres.

In this example an airplane flies at an altitude of $4km$ with a velocity of $300m.s^{-1}$ and then executes a $2g$ turn. Figure 3a shows the route measurements tracked with predictions and estimates, and Figure 3b the mode probability of the high manoeuvre filter on the right.

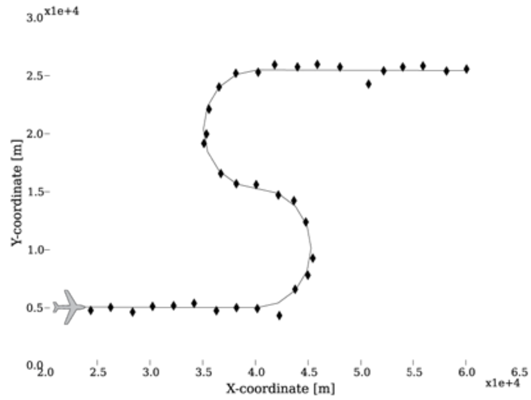


Figure 4: Test flight configuration.

Filter name	RMS(Δy)	Improvement
Kalman-4	500.602m	9.09%
Kalman-6	516.678m	6.17%
IMM	486.968m	11.56%

Table I: Comparison of filters.

3.3. *Comparison*

The performance of the Kalman and Multiple Model filters is now evaluated by comparing the root mean square error of the estimated position. At time step i the root mean square of difference between the true position y_i and the position estimate $\hat{y}_i$ is taken given by Equation 12.

$$RMS(\Delta y) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{12}$$

Figure 4 is an example of a flight configuration that contains straight sections and two tight $5g$ turns. The results are given in Table I, the improvement listed is the percentage it improves from the RMS error of the raw observation. The Kalman 6-state generally performs better on turns, but does not perform well in straight sections. The IMM filter mixes between a low process noise 4-state and a high-process noise 6-state filter, and in this case it performs the best. For other test configurations it also scores consistently higher, and has superior capability to cope with different kinds of motion.

4. TRACK MANAGEMENT

An estimation filter alone would be sufficient for the tracking of a single object under ideal conditions, but the noisy nature of the measurements can make the correct association a difficult task when multiple aircraft interact closely. This is made even more difficult when considering the additional challenges faced by a radar system such as false and missing reports, new targets and targets that end.

4.1. *Multiple Hypothesis Tracker*

The Multiple Hypothesis system manages a collection of hypotheses, each hypothesis representing a situation of possible targets and paths that could have been taken. So each hypothesis consists of a collection of tracks, where a track is a sequence of measurements and missed detections that represent the possible movement of an aircraft.

For each scan of measurements, the MHT looks at each of the existing hypotheses and creates new hypotheses for every possibility of track-measurement association. Missed detections, false targets and new targets are handled as well.

The next step is to estimate and predict the tracks. Each track of a hypothesis is represented by an estimation filter (Kalman or otherwise), and each one is updated according to the previously associated measurement.

Now the probability of a hypothesis is updated according to the measurement association and its nature. After this the hypotheses are compared, and those that are less likely are removed. At this moment the system awaits the next batch of measurements to restart the cycle.

4.2. *Theory*

Following Blackman [5], the probability that hypothesis K happened is given by:

$$\begin{aligned} Q_K &= C\beta_{FT}^{n_{FK}}\beta_{NT}^{n_K}\prod_{i=1}^{n_K}\Big[P_{TL}(D_i)P_D^{N_i} \\ &\quad (1-P_D)^{D_i-N_i}\prod_{j=1}^{N_i} f(z_i(j))\Big] \end{aligned} \tag{13}$$

where the algorithm is described below.

- $\beta_{FT}^{n_{FK}}$, $\beta_{NT}^{n_K}$: The sources of the tracks. An assumptions is made that targets arise randomly in space with uniform probability densities. β_{FT} represents the density for false targets, and β_{NT} for new targets. These densities are compounded for the false targets n_{FK} and the true targets n_K.
- $P_{TL}(D_i)$: The likelihood of a track disappearing from the search volume given the track length D_i.
- $P_D^{N_i}$, $(1-P_D)^{D_i-N_i}$: The probability of detection P_D (an aspect specific for a radar system) is compounded for the N_i detections of the track and the remainder $(1-P_D)$ for the $D_i - N_i$ missed detections.
- $f(z_i(j))$: The main contribution to the probability of a hypothesis is to what extent the observation $z_i(j)$ associate successfully with the track i. Finding this probability we use a multivariate Gaussian distribution to describe the probability density function of the residual error (the difference between the predicted and the real measurements).

The innovation covariance S and the observation prediction $\hat{z}$ of the track's Kalman filter is given by:

$$\begin{aligned} S &= HPH^T + R_c \\ \hat{z}(k \mid k-1) &= H\hat{x}(k \mid k-1). \end{aligned} \tag{14}$$

The difference between the predicted and real observation is given by:

$$\tilde{z}(k) = z(k) - \hat{z}(k \mid k-1). \tag{15}$$

The density function is evaluated with the observation as input:

$$f(z) = N[z(k); \hat{z}(k|k-1), S] = \frac{e^{-\tilde{z}^T S^{-1}\tilde{z}/2}}{(2\pi)^{D/2}\sqrt{|S|}}. \tag{16}$$

4.3. *Example*

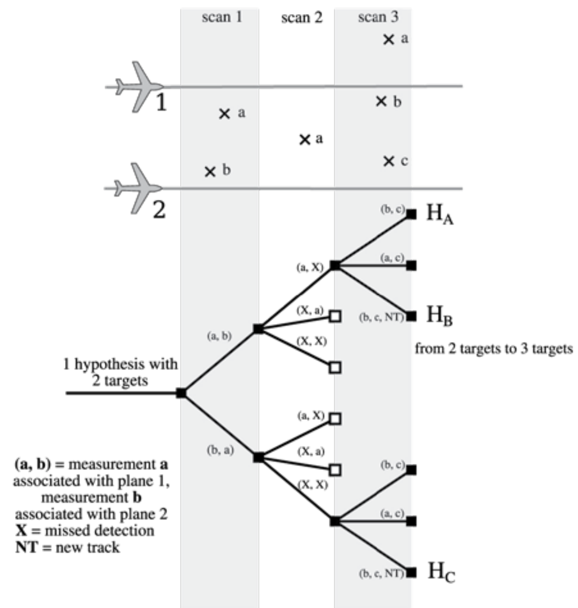


Figure 5: Example of hypothesis branching over time.

$$\begin{aligned} P(H_A) &= \beta_{nt}\beta_{ft}P_D^5(1-P_D)\prod g_{A1..A5} \\ P(H_B) &= \beta_{nt}^2 P_D^5(1-P_D)\prod g_{B1..B5} \\ P(H_C) &= \beta_{nt}^2\beta_f P_D^4(1-P_D)^2\prod g_{C1..C4} \end{aligned} \quad (17)$$

Figure 5 illustrates the tracking of the flight of two aircraft over three sets of received measurements and Equation 17 the resulting probabilities. A single hypothesis at the start represents two airplane tracks, and at each scan the hypothesis is branched into new hypotheses. In the figure, a branching tuple (a, b) indicates that measurement a of the current scan associated with track of plane 1 and measurement b associated with track of plane 2. H_A considers observation a of scan 3 correctly as a false target, while H_B and H_C consider it the start of a new target. During scan 2 there is only one observation so a target has been missed. H_C considers that single observation as a false target.

Figure 5 is a relatively simple example, and the hypothesis tree generated is substantially more involved than the one depicted here. Culling is therefore essential to prohibit an unmanageable number of hypotheses as demonstrated at the end of scan 2.

5. IDENTITY CONFIDENCE

A tracker outputs a series of tracks each consisting of a set of associated observations, while the other remaining observations are considered false targets. The task of identity confidence estimation is to take a track, and by comparison with the remaining observations, determine the probability that its identity integrity remained preserved.

The idea is to use the MHT algorithm retrospectively and to consider all the other likely possibilities. Given the initial tracks and their endings, the MHT can reconstruct possible track associations and combine the probability of all hypotheses sharing a specific track start and ending.

This is similar to using a MHT as a tracker, but differs by giving initial tracks as input. The algorithm can be applied with more focus on an area of interest, and since real time usage is not that important in this context, it can be simulated at greater depth. In essence, the hypotheses with all the different possible variations of the initial track are compared with hypotheses where the initial track do not occur. Where $H[tracks]$ select the hypotheses containing any of the supplied tracks, and $track[z_a, z_b]$ selects the tracks starting with z_a and ending with z_b,

$$P(z_a \rightarrow z_b) = \sum P(H[track[z_a, z_b]]) \quad (18)$$

gives the probability that an aircraft moved from starting observation z_a to the suspected end observation z_b. Using the MHT in this way the algorithm can handle any radar site setup, flying configuration and manoeuvre. The simplest case of interacting aircraft is a crossing bypass flight. Two airplanes flying directly next to each other in the same direction offers no chance of track identity preservation. On the other hand, flying past each other in opposite directions no confusion should be possible. Figure 6 shows results of different bypass configurations for 100 runs each expressed in histogram format. The probability considered is the probability that the airplanes did indeed cross. With 10 degree crossing the choice between the two possibilities (as shown in Figure 1) is equally likely, while at 140 degrees confusion is considered unlikely. These results confirm intuition.

In combat situations a visual inspection is a common manoeuvre. When a aircraft of unknown identity enters the airspace, another aircraft is dispatched to identify the target. This involves close quarters manoeuvring as the inspection aircraft swoops in behind its quarry and there is good chance of target identity confusion during the manoeuvre.

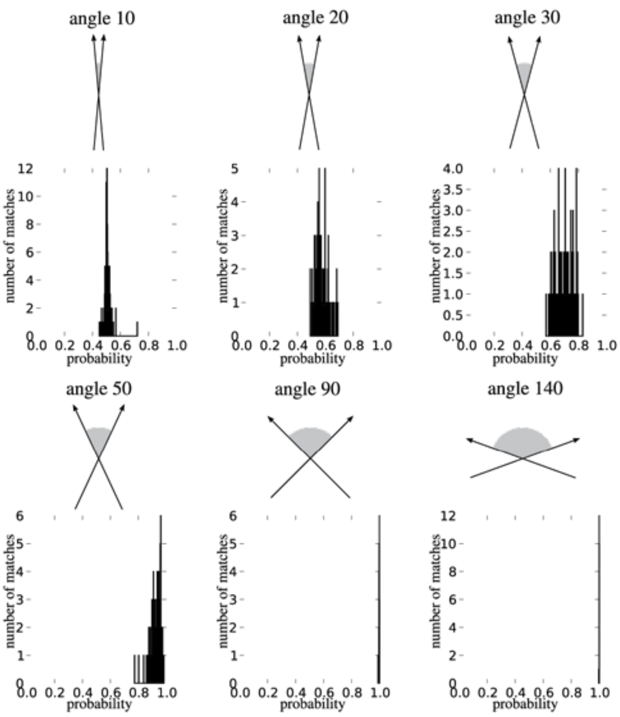


Figure 6: Identity confidence of a bypass flight.

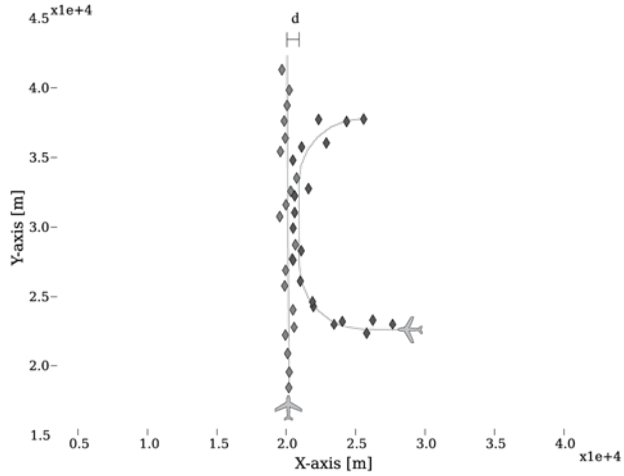


Figure 7: Inspection manoeuvre.

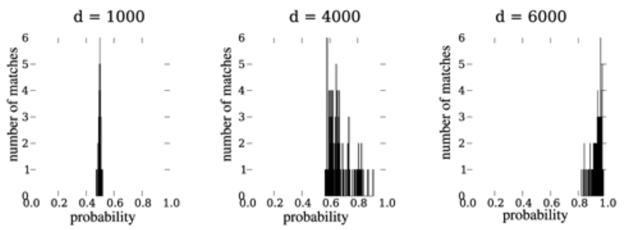


Figure 8: Identity confidences of inspection flights for various values of d.

This flight pattern is depicted in Figure 7. In this situation we will compare the probabilities obtained of paths bypasses various distances of d.

Figure 8 shows that at 1 km bypass there is not much certainty to be attached to any identity. For larger values of d the certainty rises until at the farthest bypass of 6 km the identities most probably remain preserved.

6. CONCLUSION

Posed with a problem from the industry to determine the identity confidence of radar tracking results, we have decided to approach the problem from the radar tracking methodology side. Multiple Hypothesis Tracking is a good way to extract probabilities from a scenario, and it uses numerous Kalman filters to estimate the best hypotheses. Variations of filtering were considered, and the Interacting Multiple Model filter proves to perform the best.

Extending the MHT and using it ex post facto on tracker output, we now have a robust system that can handle multiple aircraft while incorporating uncertainties of radar environment with ease. By applying the identity confidence system on simple bypass flight benchmark, the results obtained match the expected.

7. REFERENCES

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