COOPERATIVE DIVERSITY USING REPEAT-PUNCTURED TURBO CODES

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Abstract: When mobiles cannot accommodate multiple antennas due to size, hardware complexity and/or other constraints, space-time coding cannot be used to provide uplink transmit diversity. Recently, cooperative diversity has been introduced where mobiles achieve uplink transmit diversity by relaying each other's messages. A particularly powerful variation of this principle is coded cooperation which partitions the codewords of each mobile and transmits portions of each codeword through independent fading channels. Coded cooperation framework has been easily extended using turbo codes, since cooperative coding contains two codes components. It has been shown that the conventional turbo codes can be improved by repetition and puncturing. This paper presents another extension to coded cooperation. We investigate the application of repeat-punctured turbo codes in coded cooperation. The analysis and simulation results show that the proposed scheme achieves approximately 2dB gain for a 12dB inter-user channel at a BER of 10^{-4} order.

Key words: Cooperative diversity, diversity, turbo codes.

1. INTRODUCTION

The mobile wireless channel suffers from multipath fading, which causes the signal attenuation to vary significantly over a given transmission [1] and makes it extremely difficult for the receiver to determine the transmitted signal, unless the receiver is provided with some form of diversity. Transmit diversity methods have been studied extensively as a means to combat the deleterious effects of multipath fading in wireless fading channels [2]-[4]. Due to size, hardware complexity and/or other constraints, transmit diversity methods are not applicable to many wireless communication systems. For example in the uplink of a cellular communication system, where the size of the mobile is a limiting factor. However, most wireless systems operate in multi-user scenario. Cooperation among users has been suggested in [5]-[7][19][20], where mobiles share their antennas to achieve uplink transmit diversity. Diversity is achieved by transmitting a user's data through different paths. Coded cooperation proposed in [8] combines cooperative diversity with channel coding. In this scheme, cooperation occurs through puncturing a user's codeword such that part of the codeword is transmitted by the user itself while the remainder of the codeword is transmitted by the partner.

To avoid error propagation, detection is employed at the partner. An extension to the coded cooperation framework is presented in [9] where the application of turbo codes (TC) in coded cooperation is investigated. Turbo codes employ two recursive systematic convolutional (RSC) codes with interleaving [10]. The interleaver permutes the information bits within a frame in order to reduce the chance of both RSC encoders generating low weight parity sequences. The performance of TC improves with increasing interleaver size due to interleaver gain [11]. Increasing the interleaver size requires increasing the frame length since the interleaver size is identical to the information frame length in conventional TC, resulting in transmission delays. A simple but effective TC structure termed repeat-punctured turbo code is proposed in [12], which enables the use of interleavers of size larger than the information frame length. It is also shown, via distance spectrum analysis and computer simulations, that Repeat-Punctured Turbo Codes (RPTC) significantly outperform conventional TC, for moderate to high signal-to-noise ratios (SNR) in additive White Gaussian Noise (AWGN) channel.

In this paper, we propose a coded cooperation using repeat-punctured turbo codes in slow fading. We show that coded cooperation with RTPC outperforms turbo coded cooperation introduced in [9]. The remainder of this paper is organized as follows. In Section 2, we describe the system model and the performance analysis is provided in Section 3. The numerical results are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. CODED COOPERATION WITH RPTC

2.1 System Model

We consider a communication system, and focus on the cooperation between two users. This can be extended to a scenario with more than two users. The channels between the users and the destination (uplink) are independent of each other and independent of the channels between the users (inter-user). We assume that the inter-user channels are reciprocal. All channels are subject to flat fading. We consider the case in which the destination or receiver maintains channel state information and employs coherent detection so that in the analysis we need only consider the fading coefficient magnitudes. Our system uses BPSK modulation, with all users having the same transmit power. For the BPSK modulation, the baseband-equivalent discrete-time signal transmitted by user $i \in \{1,2\}$ and received by user $j \in \{0,1,2\}$ $(j \neq i)$, where $j = 0$ denotes the destination, is given by

$$r_{i,j}(n) = \alpha_{i,j}(n)\sqrt{E_{b,i}}\,b_i(n) + z_j(n), \qquad (1)$$

where $E_{b,i}$ is the transmitted energy per bit for user i, $b_i(n) \in (-1,+1)$ is the BPSK modulated code bit at time n, $\alpha_{i,j}(n)$ is the fading coefficient magnitude between users i and j, and $z_j(n)$ accounts for the effects of additive receivers noise and other forms of interference, and is modeled as zero-mean, mutually independent, white complex Gaussian random variables with variance $\sigma^2 = N_o/2$ per dimension. The fading coefficients $\alpha_{i,j}(n)$ are modeled as independent sample of Rayleigh-distributed random variable characterized by the mean-square value $\Omega_{i,j} = E_{\alpha_{i,j}}\left[\alpha_{i,j}^2(n)\right]$, where $E_x[.]$ denotes the expectation operator with respect to the random variable x. For slow fading, the fading coefficients remain constant over the transmission of each source block. The instantaneous received SNR for the channel between users i and j is defined as

$$\gamma_{i,j} = \frac{\alpha_{i,j}^2(n)E_{b,i}}{N_o}. \qquad (2)$$

For $\alpha_{i,j}(n)$ Rayleigh distributed, $\gamma_{i,j}(n)$ has an exponential distribution with mean

$$\Gamma_{i,j} = E_{\alpha_{i,j}}\left[\gamma_{i,j}(n)\right] = \Omega_{i,j}\frac{E_{b,i}}{N_j}. \qquad (3)$$

We assume the channels statistics are not changing with time, that is, $\Omega_{i,j}$ and $\Gamma_{i,j}$ are constant over n for a given channel.

2.2 Implementation Issues

The implementation of coded cooperation using repeat-punctured turbo codes is described here. Repeat-punctured turbo codes consist of two parallel concatenated component code encoders. The encoder of the first component code is similar to the recursive systematic convolutional encoder of conventional turbo codes. However, the second one is different from the RSC encoder of turbo code. In conventional turbo code, the information sequence of length N is directly permuted by an interleaver, whereas in the case of RPTC, the information sequence of length N is first repeated L times and then permuted by an interleaver of size LN [12]. For our scheme, we use a repeater $L = 2$. The encoder and decoder structures for each user are shown in Figs. 1 and 2.

The implementation of coded cooperation using repeat-punctured turbo codes is shown in Fig. 3. The codeword for the first frame is obtained using the first RSC. The user repeats twice the source bits of length N before permuting through an interleaver of size $2N$, punctures to N bits and then transmits the parity bits corresponding to the second RSC. This is done upon successful decoding of the partner.

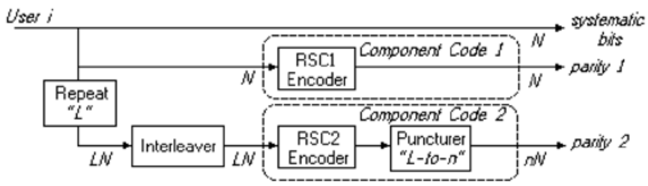

Fig. 1. RPTC encoder structure.

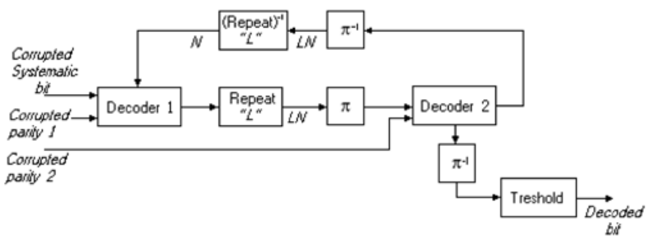

Fig. 2. Structure of the RPTC decoder.

This scheme has a fixed cooperation percentage of 33%. At the destination, the combination of the first and second frames offers the possibility of turbo decoding.

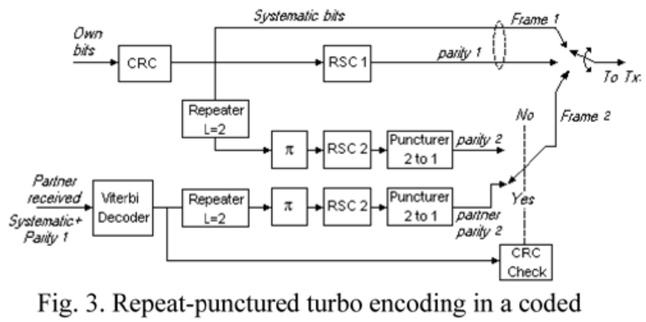

Fig. 3. Repeat-punctured turbo encoding in a coded cooperation scheme.

3. PERFORMANCE ANALYSIS

In this section, we provide an analysis of the coded cooperation with RPTC. First the pairwise error probabilities are derived by using the tools and techniques developed in [13]. This is done in Section 3-1. In Section 3-2, we derive the weight enumerators as in [16] and finally in Section 3-3, the upper union bounds for the end-to-end bit error probability are determined.

3.1 Pairwise Error Probability

The pairwise error probability (PEP) for a coded system is the probability of detecting an erroneous codeword e when codeword c was transmitted. The PEP is written as [14, (12.13)],

$$P(c \rightarrow e \mid \gamma) = Q\left(\sqrt{2\sum_{n\in\eta}\gamma(n)}\right), \tag{4}$$

where $Q(x)$ denotes the Gaussian Q-function [15]. The set η is the set of all n for which, $c(n) \neq e(n)$, so the cardinality of η, $|\eta| = d$,where d is the hamming distance between c and e. For two-user cooperation, four cases arise:

a) The case of full cooperation, that is, when both users decode each other's coded bits successfully, the coded bits are divided between the two user channels. In this case, we can write (4) as

$$P(d \mid \gamma_1, \gamma_2) = Q\left(\sqrt{2d_1\gamma_1 + 2d_2\gamma_2}\right), \tag{5}$$

where γ_1 and γ_2 denote user 1 and 2's uplink channel respectively, d_1 and d_2 are the numbers of bits in the Hamming weight d that are transmitted through user 1's and user 2's uplink channel. The unconditional probability is obtained by averaging (5) respectively over the fading distributions, as

$$P(d) = \int_0^\infty\int_0^\infty P(d \mid \gamma_1, \gamma_2)\, p(\gamma_1)\, p(\gamma_2)\, d\gamma_1 d\gamma_2, \tag{6}$$

where $p(x)$ is the probability density function (PDF) of x. The exact solution can be derived using the techniques in [14] as described in [8] and the resulting upper bound is

$$P(d) \leq \frac{1}{2}\left(\frac{1}{1+d_1\bar{\gamma}_1}\right)\left(\frac{1}{1+d_2\bar{\gamma}_2}\right), \tag{7}$$

where $\bar{\gamma}_i$ is the average SNR for user i's uplink channel.

b) In the non-cooperation case, the conditional PEP is obtained by setting $d_1 = d$ and $d_2 = 0$ in (5), resulting in

$$P(d \mid \gamma) = Q\left(\sqrt{2d\gamma}\right), \tag{8}$$

and the unconditional PEP becomes

$$P(d) \leq \frac{1}{2}\left(\frac{1}{1+d\bar{\gamma}}\right). \tag{9}$$

c) The case in which user 1does not successfully decodes user 2, but user 2 successfully decodes user 1, the conditional PEP for user 1 can be written as

$$P(d \mid \gamma_1, \gamma_2) = Q\left(\sqrt{2d\gamma_1 + 2d_2\gamma_2}\right). \tag{10}$$

And the unconditional PEP becomes

$$P(d) \leq \left(\frac{1}{1+d\bar{\gamma}_1}\right)\left(\frac{1}{1+d_2\bar{\gamma}_2}\right). \tag{11}$$

3.2 Weight enumerators

Transfer function bounding techniques studied in [16] were used to derive the union bounds of the repeat-punctured turbo codes. We start with the transition matrix as shown in (12)

$$A(L,I,D) = \begin{pmatrix} L^l I^i D^d{}_{0,0} & \cdots & L^l I^i D^d{}_{0,2^m} \\ \vdots & \ddots & \vdots \\ L^l I^i D^d{}_{2^m,0} & \cdots & L^l I^i D^d{}_{2^m,2^m} \end{pmatrix} \tag{12}$$

where l is always equal to 1 for rate $1/m$, i and d are either 0 or 1, depending on whether the corresponding input and output bits are 0 or 1 respectively in the monomial $L^l I^i D^d$. For a given code fragment, defined by a state diagram with 2^m states and an m bits terminated trellis path, the generating function is defined by

$$T(L,I,D) = \sum_{l\geq 0}\sum_{i\geq 0}\sum_{d\geq 0} L^l I^i D^d t(l,i,d), \tag{13}$$

where $t(l,i,d)$ denotes the number of paths of length l, input weight i and output weight d starting and ending in the state 0^m. Since $I + A + A^2 + A^3 + \cdots = (I - A)^{-1}$, the transfer function can be reduced to

$$T(L,I,D) = \left[\left(I - A(L,I,D) \right)^{-1} \right]_{0^m,0^m}. \quad (14)$$

Using the recursive method explained in [16], we can determine $t(l,i,d)$ for the $(15/13)$ code fragment used for our simulations, for $l \geq 0, i \geq 0, d \geq 0$:

$$\begin{aligned} t(l,i,d) =\; & t(l-1,i,d) + t(l-1,i-1,d-1) - t(l-6,i-5,d-5) \\ & - t(l-6,i-1,d-1) + t(l-6,i-5,d-1) + t(l-6,i-1,d-5) \\ & - t(l-7,i-5,d-1) + t(l-7,i-7,d-3) - 2t(l-7,i-5,d-5) \\ & - t(l-7,i-1,d-5) + t(l-7,i-3,d-7) - t(l-7,i-6,d-2) \\ & - t(l-7,i-2,d-6) + 2t(l-7,i-3,d-3) - 2t(l-7,i-2,d-2) \\ & + t(l-7,i,d-4) + t(l-7,i-4,d) + 2t(l-7,i-4,d-4) \\ & - t(l-8,i-8,d-4) - t(l-8,i-4,d-8) - t(l-8,i-4,d) \\ & + 2t(l-8,i-6,d-6) + 2t(l-8,i-2,d-6) + 2t(l-8,i-6,d-2) \\ & + 2t(l-8,i-2,d-2) - t(l-8,i,d-4) - 4t(l-8,i-4,d-4) \\ & + \delta(l,i,d) - \delta(l-1,i-1,d-1) - \delta(l-2,i-1,d-1) - \delta(l-3,i-1,d-1) \\ & - \delta(l-4,i-1,d-1) + \delta(l-4,i-3,d-3) - \delta(l-5,i-1,d-1) \\ & + \delta(l-5,i-3,d-3) - \delta(l-6,i-1,d-5) + 2\delta(l-6,i-3,d-3) \\ & - \delta(l-6,i-5,d-1) + \delta(l-7,i-2,d-6) - \delta(l-7,i,d-4) \\ & - \delta(l-7,i-4,d) + 2\delta(l-7,i-2,d-2) - 2\delta(l-7,i-4,d-4) \\ & + \delta(l-7,i-6,d-2). \end{aligned} \quad (15)$$

Denote the conditional probability of producing a codeword fragment of weight d given a randomly selected input sequence of weight i by (16) for the first encoder

$$p(d_1 \mid i) = \frac{t(N,i,d_1)}{\sum_{d_1} t(N,i,d_1)} = \frac{t(N,i,d_1)}{\binom{N}{i}}, \quad (16)$$

and by (17) for the second encoder

$$p(d_2 \mid i) = \frac{t(LN,Li,d_2)}{\sum_{d_2} t(LN,Li,d_2)} = \frac{t(LN,Li,d_2)}{\binom{LN}{Li}}. \quad (17)$$

3.3 Bit Error Rate Analysis

The PEP and the weight enumerators derived in Section 3.1 and Section 3.2 will enable us to compute the end-to-end bit error probabilities. We first need to calculate the probability of operating in the cooperative mode, which can be parameterized with $\Theta = \{1,2,3,4\}$, where $\Theta = 1$ corresponds to the full cooperation case, $\Theta = 2$ is the non-cooperation case, $\Theta = 3$ corresponds to the case in which user 1 does not successfully decode user 2, but user 2 successfully decodes user 1 and $\Theta = 4$ is the case in which user 1 successfully decodes user 2, but user 2 does not successfully decode user 1. For the full cooperation case, the conditional inter-user block error probability is given by

$$\begin{aligned} P(\Theta = 1 \mid \gamma) &= \left(1 - P_{block}^{(1)}(\gamma)\right)\left(1 - P_{block}^{(2)}(\gamma)\right) \\ &\leq \left(1 - P_E^{(1)}(\gamma)\right)^B \left(1 - P_E^{(2)}(\gamma)\right)^B \\ &\leq \left(1 - BP_E^{(1)}(\gamma)\right)\left(1 - BP_E^{(2)}(\gamma)\right), \end{aligned} \quad (18)$$

where $P_{block}(\gamma)$ is the block error rate which, using the method in [17] is bounded by

$$\begin{aligned} P_{block}(\gamma) &\leq 1 - \left(1 - P_E(\gamma)\right)^B && (19) \\ &\leq BP_E(\gamma). && (20) \end{aligned}$$

In (20) B is the number of trellis branches in the codeword and $P_E(\gamma)$ is the error event probability. For a $1/n$ convolutional code, B equals the uncoded block length K. It can be shown that $P_E(\gamma)$ is bounded by [18]

$$P_E(\gamma) \leq \sum_{d=d_{\min}}^{\infty} a(d) P(d \mid \gamma), \quad (21)$$

where $d_{\min}$ is the minimum distance of the code and $a(d)$ is the number of error events with Hamming distance d. The conditional inter-user block error probability for the other cases can be derived in a similar method as in (18) and is omitted here. The end-to-end bit error rate probabilities is calculated using the unconditional inter-user block error probability $P(\Theta)$, and given by

$$P(\Theta) = \int_\gamma P(\Theta \mid \gamma).p(\gamma)d\gamma. \quad (22)$$

Using (20) yields loose bounds in the case of slow fading as shown in [17]. The limit-before-average technique is used in (19) to give tight bounds on the block error probability. For the case of full cooperation, this tight upper bound is obtained by

$$\begin{aligned} P(\Theta = 1) \leq & \int_\gamma \left(1 - \min\left[1, \sum a_1(d) P(d \mid \gamma)\right]\right)^B \\ & \times \left(1 - \min\left[1, \sum a_2(d) P(d \mid \gamma)\right]\right)^B p(\gamma) d\gamma. \end{aligned} \quad (23)$$

The other case can be obtained in the same way.

The overall end-to-end BER is obtained by averaging the unconditional BER over the four possible cases discussed above, as shown

$$P_b = \sum_{i=1}^{4} P_b(\Theta) P(\Theta = i), \tag{24}$$

where P_b is the BER. The conditional BER uses a bound similar to the bound in [18, (4.4.8)]

$$P_b(\gamma, \Theta) \le \sum_{d=d_{\min}}^{\infty} a(d) P(d \mid \gamma), \tag{25}$$

where $a(d)$ is given by

$$a(d) = \underbrace{\sum_{i} \sum_{d_1} \sum_{d_2}}_{d=i+d_1+d_2} \frac{i}{k} \binom{k}{i} P(d_1 \mid i) P(d_2 \mid i). \tag{26}$$

The unconditional BER is obtained by averaging (25). As discussed above, tight bounds on the unconditional BER are obtained using the limit-before-average described in [17]. The unconditional BER is thus given by

$$P_b(\Theta) \le \int_{\gamma} \min\left[\frac{1}{2}, P_b(\gamma, \Theta)\right] p(\gamma) d\gamma. \tag{27}$$

4. NUMERICAL RESULTS

In this section, we plot some examples of the upper bounds on the unconditional BER P_b using the limit-before-average technique described in [17] and compare them with the simple average technique. We also compare the BER bounds with simulation results for the RPTC code implementation of coded cooperation. For all the simulations, we use a source block code of $N = 128$ bits. We employ a rate $1/2$, eight-state constituent codes with generator polynomial $G(1,15/13)_{octal}$ and a repeater $L = 2$. The cooperation percentage is 33%. The overall code rate is $1/3$.

The convergence of the union bound on the bit error probability using the simple average method and the one obtained from the limit-before-average method is shown in Fig. 4. It is seen that the limit-before-average union bound on the bit error probability converges rather fast, whereas the simple average method calculated as the sum as the of the average pairwise error probabilities does not. The summation in (24) where the upper limit corresponds to $d_{\max}$ is truncated such that only the error events having the total distance $d = d_1 + \ldots + d_L \le d_{\max}$ are taken into account.

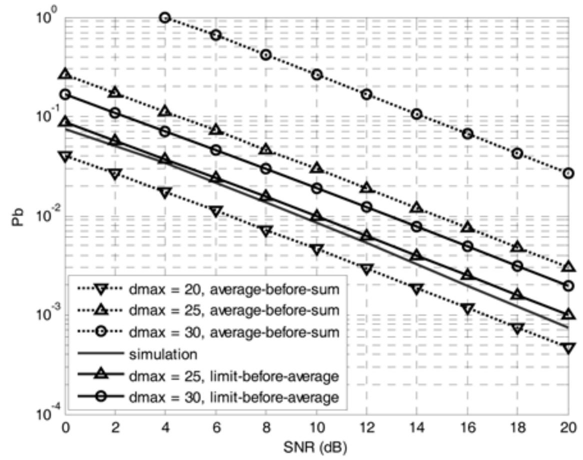

Fig. 4. Union bounds on the bit error probability P_b for different values of $d_{\max}$.

Fig. 5 compares the performance of the turbo coded cooperation [9] and the repeat-punctured turbo coded cooperation in slow fading, with an overall rate $1/3$. We note that the repeat-punctured turbo coded cooperation performs almost 2dB better than the turbo coded cooperation for higher uplink SNR, both for 6dB and 12dB inter-user channel.

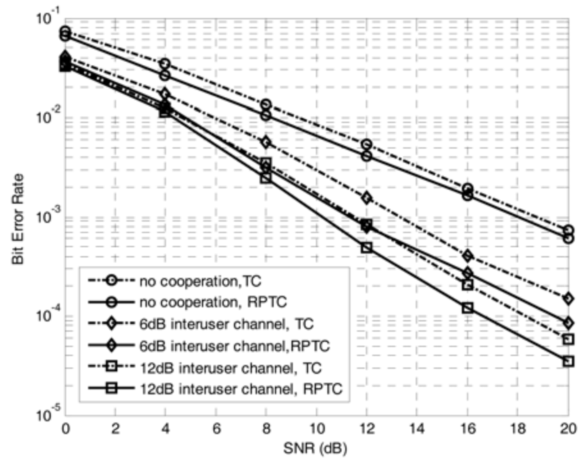

Fig. 5. Comparison of turbo coded cooperation and repeat-punctured turbo coded cooperation in slow fading.

Fig. 6 shows the theoretical BER bounds versus computer simulations for the repeat-punctured turbo coded cooperation scheme in slow fading with reciprocal inter-user channels of various qualities. The level of cooperation is 33% and the users have the same uplink channels. Combining [16] and [17] yields tight bounds on the BER as shown in Fig. 6.

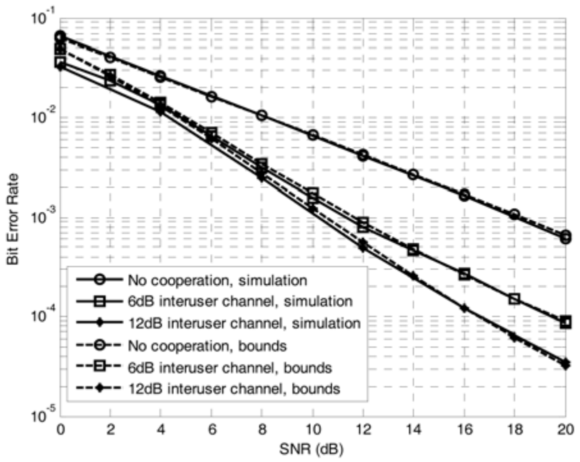

Fig. 6. Performance in slow Rayleigh fading with 33% cooperation, equal uplink SNR and reciprocal inter-user channels of the repeat-punctured turbo code with coded cooperation.

5. CONCLUSION

In this paper, cooperative diversity with repeat-punctured turbo codes is presented. Simulation results of this scheme are discussed in Section-4 as well as the theoretical bounds on the bit error probability. Prior to that, a comparison of the turbo coded cooperation developed in [9] and the repeat-punctured turbo coded cooperation is presented. It is noted that the new scheme outperforms the turbo coded cooperative scheme by almost 2dB for higher uplink for different qualities of the inter-user channel. The theoretical bounds of the RPTC with coded cooperation closely match the simulation results.

6. ACKNOWLEDGEMENTS

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