AB INITIO FREQUENCY MEASUREMENT AND CHARACTERISATION OF FREQUENCY DOUBLED FIBRE LASER UTILISED FOR PRECISION OSCILLATORS

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Abstract: An *ab initio* measurement of a free running, frequency doubled erbium fibre laser is made with an optical frequency comb in concert with an internally calibrated wavemeter. The measurement is validated via a Monte Carlo uncertainty analysis. The operation and characteristics of a new high performance optical metrology source for an optical frequency standard is also verified in the process.

Key words: Optical frequency standards, optical frequency comb, metrology, photonics.

1. INTRODUCTION

The National Metrology Institute of South Africa (NMISA) is starting to develop precision atomically-referenced optical oscillators for metrology and technological applications. Stabilised optical oscillators offer high precision, exceeding those of traditional microwave standards like the common caesium (Cs) and hydrogen standards, and also have application in ultrahigh stability RF oscillators when down converted with an optical frequency comb [1]. Optical frequency combs can be seen as an optoelectronic gearing system to convert the optical oscillators with frequencies around typically 150 THz to 600 THz to RF frequencies in the 10's of MHz range (or vice versa), while retaining the same fractional uncertainty of the frequency during the process. It can also be used to optically synthesise a frequency comb from a microwave reference for measurement and other uses, with basically the same fractional frequency accuracy, as the microwave reference.

Optical oscillators can use inherently stable lasers, to access transitions in atoms for referencing/stabilising to provide simple, but highly stable molecular clocks (like iodine stabilized green laser based on Nd:YAG or Yb-fibre technology[1, 2]). Furthermore more exquisite optical clocks utilise lasers that access ultrahigh Q transitions in atoms/ions to provide fractional frequency uncertainties $<10^{-17}$ [3]. Furthermore such oscillators offer significant advantages in distribution of frequency (RF or optical) over low loss fibre [4] or high performance antenna arrays [5], and have many other industrial and reference measurement applications.

The NMISA is developing its optical frequency systems on a robust and compact optical telecommunications fibre technology platform coupled with advanced electronics. In the last decade high performance single polarisation single frequency fibre lasers have become available in two forms, namely distributed feedback (DFB) lasers [6] and distributed Bragg reflector (DBR) lasers [7] that both use in-fibre Bragg grating technology. These lasers can interface with components originally developed for the optical fibre telecommunications field, which are usually quite robust, and include modulators, couplers, amplifiers, Faraday rotators and in-fibre filters. All of these components are spliced together in systems with a semi-automated fusion splicer, and results in a manufacturing process quite similar to the soldering together of electronic components on a circuit board. Even atomic cells can now be built in optical fibre [8]. This fibre and advanced electronics technology platform therefore provides the opportunity to build very robust, compact, transportable and potentially cheap systems for certain niche markets, which are sensitive to these attributes.

The work described in this paper is concerned with measurements on some parts of the optical metrology source subsystem that is currently being developed. The optical metrology source subsystem contains many parts, but two of the important parts are a DBR fibre laser which is frequency doubled in an external nonlinear waveguide that gives a near-infrared line. Some of the power and wavelength characteristics of this source are presented. Furthermore the frequency characteristics that is measurable on a Ti:Sapphire (Titanium doped Sapphire) laser optical frequency comb together with a wavemeter is shown. In the process it is shown that absolute optical frequency measurements can now be undertaken at the NMISA, potentially down to the ~Hz level (with the proper microwave reference) at optical frequencies of ~400 THz, without prior knowledge of the optical frequency, as an input to the process.

2. METHODOLOGY

Optical frequency combs [9] are now the standard way to link optical frequency measurements with established microwave references. Nonetheless there are now only three labs that operate such combs in the Southern Hemisphere (Brazil, Australia and South Africa) according to our knowledge, due to the relatively high complexity and cost. New fibre combs [10] have now started to make the technology quite robust though.

The comb is synthesized via a mode-locked laser and uses a highly nonlinear photonic crystal fibre for spectral broadening to reach the required spectral bandwidth of an octave. The measurement of an unknown laser (unit under test or UUT) with frequency f_{UUT} is done by detecting a beat with a beat frequency f_b against an N^{th} comb element of the optical frequency comb with frequency

$$f_N = N f_{rep} \pm f_0 \qquad (1)$$

where N is the comb element number, f_{rep} is the laser repetition rate and f_o is the offset frequency. f_{rep} is changed via the cavity mirror spacing on the laser's Fabry Perot cavity and f_o via the laser's intracavity dispersion via translation of thin glass prisms inside the cavity. If all the abovementioned quantities are known and measured the UUT's frequency is therefore simply determined to be

$$f_{UUT} = N f_{rep} \pm f_o \pm f_b \ . \qquad (2)$$

Therefore five parameters have to be known to determine the UUT's frequency, that is N, f_{rep}, f_o and f_b and the two signs of the last numbers. In practice N is the hardest to determine and is the point of discussion in the next 3 sections.

2.1 Resolving comb elements with comb only: General remarks

Resolving the unknown comb number N in such optical frequency measurements is still challenging due to a typical close comb element spacing of 200 MHz at optical frequencies. In order to avoid ambiguity in the measurement of N an implicit frequency pre-knowledge of the laser frequency below <0.25 parts per million (ppm) is implied at a wavelength of λ~800 nm, when a single direct measurement is undertaken. There has been some work to increase the laser repetition rates in both solid state and fibre lasers via shorter laser cavities [11], high harmonic modelocking [12] or frequency filtering in Fabry Perot cavities [13], but repetition rates around the ~200 MHz is still dominating for a range of practical reasons.

In practise f_o and f_{rep} is controlled via proportional-integral (PI) feedback control electronics in the Menlo Systems comb (model FC-8004), to ensure f_b is within the detection bandwidth of bandpass electronics in the system (a few MHz at ~30 MHz). The abovementioned frequency comb, which is also utilised at the NMISA has a repetition rate of around 200 MHz, and both the repetition rate and offset frequency are referenced to a Cs frequency standard (Agilent 5071A). f_o is fixed to an absolute value of 20 MHz. This frequency fixture limit on f_0 and f_b and the adjustment limitations of f_o and f_{rep} therefore constrains the operable parameter space of the instrument, but with an advantage of sensitive detection due to component optimization. Alignment of ultrashort pulses in propagation vector, polarization, wavelength, and time domains is necessary to detect the offset beat f_o. Typically the UUT vs. comb beat is quite weak because it involves the mixing of an external laser (UUT) with typically ≤1 mW power (else the avalanche photodetector will saturate) with a comb element that has only a few tens of nW in power. Nevertheless, with careful alignment the latter UUT vs. comb beat will result in a ~ 30 dB signal to noise ratio that is typically necessary for successful frequency counting. Furthermore the sign of f_b is determined by slightly changing the repetition rate, while observing the change in beat frequency, f_b. The sign of f_o is changed by adjusting the intracavity dispersion.

2.2 Procedures for resolving comb elements with comb

A "simple" procedure that could theoretically resolve the number N, which has been attempted at the NMISA, is detailed here to give an idea of some of the difficulties that are encountered in resolving the exact frequency. This procedure works by keeping the comb element number N constant, but tuning f_{rep} so that f_o can alternate between positive and negative.

Assume that the frequency of the unknown laser, f_{UUT}, stays constant between performing two sets of measurements; one set with the offset frequency, f_0, tuned to be negative and the second with the offset frequency positive. During the measurements, care was taken to keep the comb element, N, the same for both measurements.

$$f_{UUT} = N f_{rep1} - f_{01} + f_{b1} \qquad (3)$$

$$f_{UUT} = N f_{rep2} + f_{02} + f_{b2} \qquad (4)$$

Since both the laser frequency and the comb element are unknown, one must be eliminated from the two equations. By setting Equations 3 and 4 equal, and adding uncertainty contributors (assuming dominant uncertainties are coming from the UUT vs. comb beat notes) one obtains

$$N + \Delta(N) = \frac{\left(f_{01} + f_{02} + f_{b2} - f_{b1} + \Delta(f_{b2} - f_{b1})\right)}{f_{rep1} - f_{rep2}} \ , \qquad (5)$$

where $\Delta(x)$ denotes the uncertainty in a number x. But the uncertainties in f_{b2} and f_{b1} have the same Gaussian-like distribution, and therefore uncertainty in the difference is $\Delta(f_{b2} - f_{b1}) = \sqrt{2}\,\Delta(f_{b2}) = \sqrt{2}\,\Delta(f_{b1}) = \sqrt{2}\,\Delta(f_b)$ [14]. From Equation 5 the uncertainty in N is therefore

$$\Delta(N) = \frac{\sqrt{2}\Delta(f_b)}{f_{rep1} - f_{rep2}}, \tag{6}$$

where the difference between the two repetition frequencies $f_{rep1} - f_{rep2}$ is small; typically ~ 20 Hz. To obtain $\Delta(N) < 0.5$ with 95% confidence, the uncertainty (standard deviation) in individual beat frequency measurements must be < 4 Hz. Some of the lasers, as discussed in this paper have uncertainty even up to the MHz level on the timescales of the measurements, and therefore this methodology becomes unfeasible for a large number of lasers utilised within metrology environments. It should be noted that a two-step method has been suggested and tested by others for measurement of lasers with up to 10's of kHz of uncertainty [15], but this still falls short for really noisy lasers (some of which might be quite accurate though).

2.3 Method for measurements at the NMISA

So far these measurement procedures that use the comb only for frequency information could not be fully implemented at NMISA, especially as lasers with relatively large uncertainties were being measured. These uncertainties stem from processes that include frequency modulation of the UUT that is used to lock the laser to an atomic/molecular cell (like on older national length standards), or other intrinsic noise or drift of the lasers. Nevertheless accurate average values are needed on some of these sources as some of them are even used in real metrological setups (for example for dimension) where traceability to primary time standards is necessary. Therefore some initial rough measurement is needed, especially for the lasers that have been used so far, to obtain an estimate on the comb mode number for the subsequent frequency comb measurement.

Therefore a wavemeter with an internal laser reference is utilised at the NMISA for performing a premeasurement. Previous attempts at using a non-referenced Fabry Perot wavemeter were unsuccessful; even these meters drift over time and smaller than 0.25 ppm absolute accuracy is needed at all times. This need is now fulfilled by utilizing a 0.2 ppm absolute accuracy wavemeter from Bristol Instrument (series 621). This wavemeter has an internal stabilized Helium Neon laser for continuous self-calibration. It was necessary to show though that the wavemeter could indeed resolve the actual comb element numbers in a statistically significant way to have confidence in the measurement process and effectively validating it. A new metrology source (as under development at NMISA) was utilised in the process, and these measurements are discussed in the next section.

3. MEASUREMENTS ON DOUBLED LASER

The experimental setup is shown in Figure 1 below. The modelocked laser system produces ~50 fs slightly chirped pulses, which are injected in highly nonlinear fibre to generate combs spanning more than an octave. There are two separate supercontinuum combs – one is passed through the nonlinear interferometer to generate f_o and the other is used for measuring f_b. The two separate but coherent combs enable one to separately optimise the magnitude of the offset frequency beat and UUT vs. comb beat. The device under test was a single longitudinal mode DBR optical fibre laser (NP Photonics series RFLM) made in erbium fibre that is fully fibrerised with polarisation maintaining (PM) fibre. This source is injected into a frequency doubling waveguide after an attenuator. The connectors A and B were disconnected at various stages of the experiment to monitor power on calibrated InGaAs or Si power measurement heads (Ophir), to observe the spectra on a scanning grating spectrometer (Anritsu) or to measure the wavelength (Bristol Instruments 621).

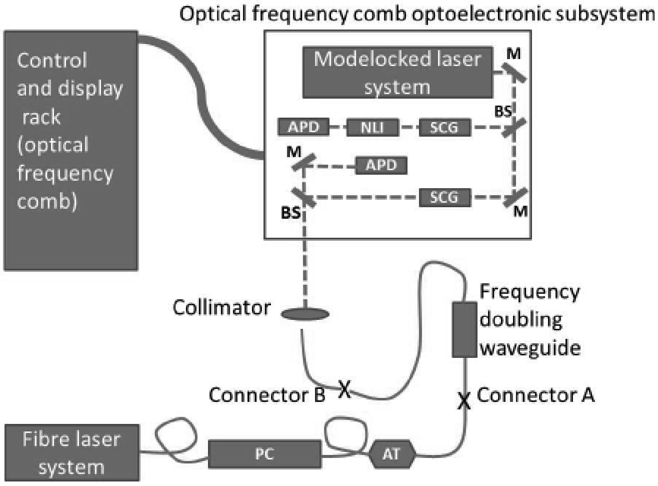

Figure 1: Simplified schematic of experimental setup. The abbreviations are as follows: PC=Polarization controller/adapter, AT=Manual attenuator, BS=Beam splitter, M=Mirror, APD=Avalanche Photodetector, SCG=Supercontinuum Generator (consisting of launching and collimating optomechanics and highly nonlinear fibre), NLI=Nonlinear Interferometer

3.1 Characteristics of frequency doubling waveguides

A custom-made periodically poled lithium niobate (PPLN) doped waveguide with MgO (the MgO increases the photorefractive damage resistance) has been used to frequency double the light from the fibre laser. The waveguide has been pigtailed with appropriate PM fibres. Such MgO:PPLN waveguide devices have been found to be convenient and efficient frequency doubling devices [16]. Such nonlinear waveguides are also used in systems for laser stabilisation to atoms [17]. The poling period is optimized for the particular laser utilised in this specific case. The waveguide device is able to generate more than

10 mW of output power for a source input power of ~140 mW (measured at the ends of the pigtails). The input wavelength is around ~1550 nm and the doubled output at half the input wavelength. This doubled wavelength is weakly visible, and therefore also convenient for length. One also expected a quadratic dependence of output power on input power as shown in the measurement of Figure 2.

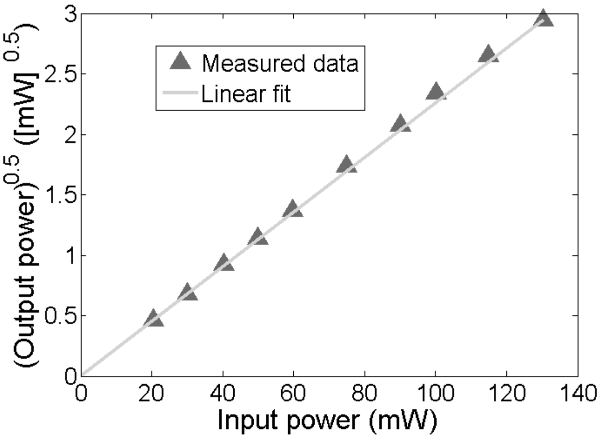

Figure 2: Output power characteristics of the MgO:PPLN waveguide, when the temperature has been optimized

Furthermore the waveguide was determined to have a wavelength acceptance bandwidth (full-width at half maximum) of ~0.05 nm. The temperature tuning characteristic is quite critical and is shown in Figure 3, which displays the nearly sinc-type shape expected from phase matching considerations.

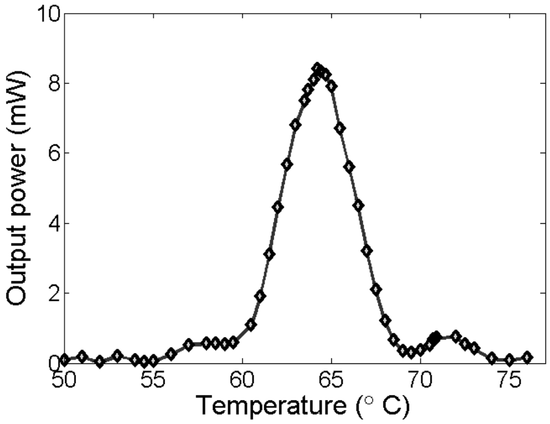

Figure 3: Temperature tuning characteristics of the MgO:PPLN waveguide

3.2 Measurements of laser against optical frequency combs

The frequency doubler places the telecommunications wavelength of the DBR fibre laser within the operational frequency range of the Ti:Sapphire laser optical frequency comb. The doubler therefore links the telecommunication range with such a visible comb that is also utilised at the NMISA and elsewhere for visible length standard characterisation and calibration.

Three separate measurements of the UUT were done over short periods of time, and the averaged frequencies recorded on the comb every 1 second. The Allan deviations [19] of these records are shown in Figure 4; some periodicity in the UUT frequency is revealed in these and the actual frequency plots. The fractional frequency deviation over the measurement times were always less than 5×10^{-9}.

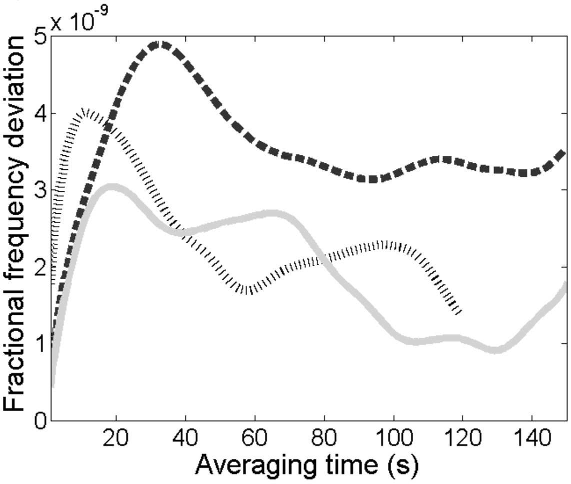

Figure 4: Overlapping Allan deviation of the UUT

Furthermore, connector B was also loosened and the fibre also intermittently connected to the wavemeter. The timerecord was also collected for the wavemeter, and an average value calculated. The corresponding sequential measurement of the laser frequency as recorded on the wavemeter and the frequency comb were always less than five minutes apart in time. The mode number was then calculated as

$$N = \left[\frac{\left(\langle f_{UUT} \rangle \pm \langle f_b \rangle \pm f_0 \right)}{\langle f_{rep} \rangle} \right] \quad , \qquad (7)$$

where the large square brackets signifies rounding and the triangular brackets averaging. f_{UUT} is determined from the wavemeter reading. The appropriate signs that were measured in each case were also applied. The offset from integer numbers (δN) were also calculated. The largest absolute value of δN was ~0.16, which means all estimated mode numbers were apparently quite accurate. The apparent accuracy is verified in the next paragraph.

3.3 Statistical interpretation of measurement and validation of measurement process

The frequency deviations of the three separate runs were combined to calculate a histogram of actual frequency deviation. The histogram was fitted with a Gaussian probability density function (PDF) for subsequent calculations. It was assumed furthermore that the extracted standard deviation was applicable to a 1 s measurement period, as is used by the electronics of the comb. The instability originating from the Cs clock that supplies referencing to the comb was accounted for by assigning a standard deviation of 2×10^{-11} to it within a 1 s averaging (measurement) time. One can now calculate a numerical PDF for δN (with symbol $\delta\tilde{N}$) when the Cs clock and laser instability and wavemeter accuracy are taken into account. In such a calculation we define the PDF of the offset from integer comb numbers as

$$\delta\tilde{N} \equiv \left(\frac{(F_{UUT} \pm F_B \pm f_0)}{F_{rep}}\right) - \left\langle\frac{(F_{UUT} \pm F_B \pm f_0)}{F_{rep}}\right\rangle. \quad (8)$$

where the second term in triangular brackets is an average over infinite time, that results in an integer number. F_{UUT} is a random variable with square PDF [18] with an average of $\langle f_{UUT}\rangle$ and standard deviation determined by the manufacturer's specification on the wavemeter. F_B is a random variable with a Gaussian PDF, an average of $\langle f_b\rangle$ and a standard deviation determined from the Gaussian fit described above. Lastly, F_{rep} is a random variable with a Gaussian PDF, an average of $\langle f_{rep}\rangle$ and an uncertainty dictated by the Cs clock. The offset frequency also has a negligible uncertainty (<1 Hz) that is not taken into account. A Monte Carlo calculation utilising ten million pseudo-random numbers for each PDF then results in the histogram shown in Figure 6 for a 1 s measurement.

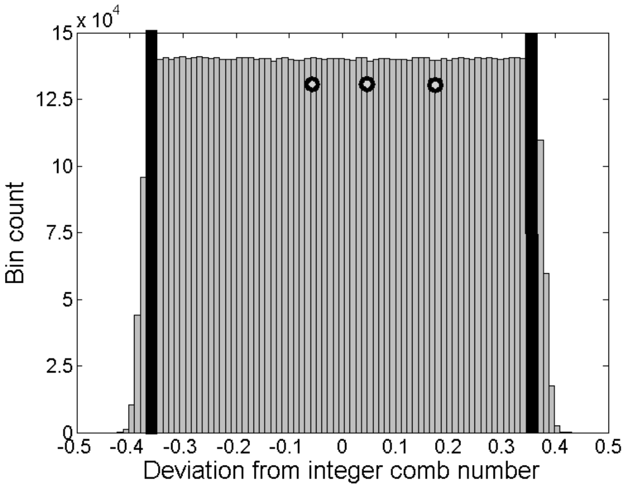

Figure 6: The numerical histogram for offset from integer comb numbers. The round dots represent averaged measured (offset) numbers. The two thick black vertical lines represent the 95% confidence interval.

4. DISCUSSION

It is clear that the wavemeter can give measurements of frequency that nearly coincides with integer comb numbers. All the measurements of comb number lies within the 95% confidence interval of the PDF of the offset from the integer mode number. One can also deduce from Figure 6 that there is a >99% probability to always get $|\delta N|<0.5$, and therefore to resolve the comb number N. Therefore, there is a high certainty that the presented methodology is working correctly in measuring frequency, because the wavemeter is used to assign real numbers to the "photonic vernier's tickmarks". The wavemeter can do this as it can measure with <0.2 ppm absolute accuracy, which allows it to operate below the ~200 MHz ambiguity range of the optical frequency comb. Furthermore the wavemeter has been calibrated by the manufacturer against other absolute standards, so that the absolute wavelength of the source can also be assured to be correct. It can also be extrapolated that it would be possible to take *absolute* frequency readings at the ~170 Hz level for more stable lasers oscillating at hundreds of THz given the ~7.5 kHz uncertainty of the comb in 1 s, and when averaging is done over a time period >½ hour (limited by the Cs reference clock). This measurement capability can of course be improved with improved references like hydrogen masers or Cs fountain clocks.

5. CONCLUSION

This paper related the first *ab initio* measurement of an unknown optical frequency utilising the Ti:Sapphire-laser based optical frequency comb at NMISA. This measurement methodology can be successfully applied to relatively noisy lasers specifically. This methodology required the use of an internally calibrated high precision wavemeter. The measurement method was shown to be statistically sound via a Monte Carlo analysis. This successful measurement is in contrast to comb-only-methods, which might end up with large uncertainties on the laser frequency values due to some uncertainty in the precise comb element number (N) value. Measurement methods that utilise the comb only, given repetition rates of ~ 200 MHz is not usable for more "noisy" lasers. Future measurement will include an extra beamsplitter and free space to fibre optic coupler, so that the comb and wavemeter measurement can take place at the same time.

The optical source was shown to be able to link the optical telecommunications range, with the near-infrared range coinciding with the input operational range of 500 nm - 1200 nm of the optical frequency comb. Because of the visual visibility of the beam it can be used in length applications. When the optical source is stabilised it will also be usable as an ultrastable oscillator for a range of time and frequency applications. In the future these improved characteristics will be measured with the same methodology as presented here, except that the Cs clock might be augmented with a hydrogen clock, to be able to measure the correspondingly higher stability.

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